Geomechanical modeling of induced seismicity source parameters and implications for seismic hazard assessment

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ABSTRACT

We simulate induced seismicity within a geothermal reservoir using pressure-driven stress changes and seismicity triggering based on Coulomb friction. The result is a forward-modeled seismicity cloud with origin time, stress drop, and magnitude assigned to each individual event. Our model includes a realistic representation of repeating event clusters, and is able to explain in principle the observation of reduced stress drop and increased $b$-values near the injection point where pore-pressure perturbations are highest. The higher the pore-pressure perturbation, the less critical stress states still trigger an event, and hence the lower the differential stress is before triggering an event. Less-critical stress states result in lower stress drops and higher $b$-values, if both are linked to differential stress. We are therefore able to establish a link between the seismological observables and the geomechanical properties of the source region and thus a reservoir. Understanding the geomechanical properties is essential for estimating the probability of exceeding a certain magnitude value in the induced seismicity and hence the associated seismic hazard of the operation. By calibrating our model to the observed seismicity data, we can estimate the probability of exceeding a certain magnitude event in space and time and study the effect of injection depth and crustal strength on the induced seismicity.

INTRODUCTION

Injecting fluid at high pressures into a reservoir rock formation reduces the effective stress and can lead to brittle failure (Terzaghi, 1923; Healy et al., 1968; Hsieh and Bredehoeft, 1981; Zoback, 2007). This creation or reactivation of fractures enhances access to the reservoir fluids (oil, gas, or hot water) and is typically associated with microearthquakes, which, if recorded and located, can give important clues about the properties and extent of the treated reservoir volume, as well as about the effectiveness of the treatment (e.g., Pearson, 1981; Warininski et al., 1995; Shapiro et al., 2002; Maxwell et al., 2009; Wessels et al., 2011). If the injection pressure exceeds the minimum principal stress $\sigma_3$ in the reservoir, tensile opening can occur and new fractures are created, which need to be kept open after the treatment with the help of a suitable “propellant” (typically sand, see Economides and Nolte, 2000). For fluid pressures lower than $\sigma_3$, failure occurs predominantly through shearing along preexisting fractures. In this case, permeability is enhanced mainly through the mismatch of rough fracture surfaces after slipping (“self-propping,” see Brown and Bruhn, 1998). In both cases, the observed seismic emission is mainly from shear slip events (e.g., Rutledge and Phillips, 2003), either by reactivation of preexisting fractures (Pearson, 1981), or through shear stress near the tip of newly created tensile fractures (Sneddon, 1946) that can extend into the host rock beyond the opened hydrofrac (Evans et al., 1999). Such seismically active shear fissures may be subject to tensile opening themselves as the pressure front advances, thus creating a network of fractures and fluid pathways in the reservoir (Hopkins et al., 1998).

Although the initiation of slip on a fracture is fairly well understood, it is very unclear what properties and processes are limiting the slip once it is initiated, and hence govern the size of a rupture area. The size of the rupture area defines the magnitude (Kanamori and Anderson, 1975), and hence the seismic energy release of a microearthquake (Kanamori and Brodsky, 2004).

On the one hand, the size of the rupture area can play a role in the increase of reservoir permeability and/or drainage efficiency. On the other hand, there is a small, but sometimes non-negligible risk that microseismic events of a noticeable strength are created that might be felt at the surface, or even cause damage (Deichmann and Gardini, 2009; Hitman et al., 2012). It therefore needs to be ensured that the magnitude of induced seismic events does not exceed values where shaking can affect surface infrastructure, and at the same time
ensuring that the economic objective of the stimulation is met. We present in this paper an approach to forward model the spatio-temporal variation of microearthquake source properties. The model can explain observed radial variations of stress drop and earthquake size statistics. Through proper calibration to observed seismicity, the approach can provide estimates of the probability of exceeding a certain magnitude event. The latter capability may be used in advanced traffic light systems to mitigate the seismic risk of injection operations.

Propagation of the microseismic event cloud through the medium can be described by the diffusive process of pore-pressure relaxation (Talwani and Acree, 1984). The hydraulic diffusivity of the medium can be estimated from the time-distance dependence of the seismicity triggering front (Shapiro et al., 1997, 2002). Apart from the spatio-temporal behavior of seismicity, source properties of induced seismicity appear to be also influenced by the hydraulic medium properties. Using data from a geothermal operation in Basel, Goertz-Allmann et al. (2011) showed that the Brune stress drop of induced events is significantly lower near the injection point and inversely correlates with estimated pore-pressure values. Similarly, Michelet (2002), Bachmann (2012), and Bachmann et al. (2012) measured the spatial variation of Gutenberg’s b-value in seismicity clouds of geothermal injections and highlighted several cases with b-values higher near the injection point where pore pressures are higher. The b-value describes the slope in the Gutenberg-Richter power law distribution, \( \log(N) = a - bM \), where \( N \) is the cumulative number of events, \( M \) is the earthquake magnitude, and \( a \) denotes the \( a \)-value describing the productivity. Seismic source property observations such as the latter contain information about the in situ stress regime: The \( b \)-value is linked to the differential stress (Amiratino, 2003; Schorlemmer et al., 2005; Gulia and Wiemer, 2010), and stress drop denotes the difference in shear stress on the fault plane before and after the earthquake (e.g., Kanamori and Brodsky, 2004).

A variety of approaches exist to forward-model the seismic response of a stimulated reservoir. For example, Rothert and Shapiro (2003) developed a numerical model to trigger microseismicity due to the process of pore-pressure relaxation and statistically predefined critical zones in the medium. Their model is able to describe the spatiotemporal distribution of induced seismicity reasonably well, but it does not include any event size distribution or other source parameters such as stress drop. A similar approach is used by Langenbruch and Shapiro (2010) to investigate the behavior of postinjection seismicity and the approach by Hummel and Müller (2009) considers nonlinear pore fluid pressure diffusion. Schoenball et al. (2010) find that the influence of hydraulic properties and the coupling of pore pressure to the stress field have an effect on the occurrence of microseismicity, but again they do not concentrate on event magnitudes. Kohl and Megel (2007) use a finite-element approach to model the hydromechanical response of the medium. Their approach attempts to model the opening or shearing of discrete (but stochastically distributed) fractures in the medium. It can take into account a Coulomb failure criterion to associate seismicity with shearing, and hence model location and time of seismic events. However, it cannot predict seismic source properties such as magnitude and stress drop. Another study also includes the opening of stochastically distributed fractures due to fluid-flow-induced stress changes to obtain a microseismicity distribution (Bruel, 2007). This study also obtains a magnitude of each event from the slip and the rupture area, but the magnitude distribution is basically predefined by the fracture distribution where each fracture can only fail completely. Baisch (2009) and Baisch et al. (2010) present an approach to physically model event sizes (magnitudes) by stress transfer across elemental slip patches. Their approach also considers a non-stationary permeability, but is restricted to individual fracture planes. Assuming a single planar fracture (which may be the dominant fluid conduit) may be inconsistent with a pressure diffusion approach (Jung, 1989). The restriction of the seismicity to a plane leads to an over-representation of modeled seismicity along the triggering front and an unrealistic representation of the seismicity back front, the so-called Kaiser effect, which may be described by an absence of seismicity due to stress relaxation (Kaiser, 1950). With increasing time, the isobar of the triggering front covers a larger circle on the fault, hence increasing the number of events along that isobar. McClure and Horne (2011) propose a new stimulation mechanism based on coupling fluid-flow with a rate-and-state friction model. This model could, in principle, also be used for hazard mitigation. However, its plausibility has not yet been fully tested against field observations. As an alternative physical modeling approach, particle-based lattice solid models (e.g., Hazzard and Young, 2002, 2004) would, in principle, be capable of simulating the effects that we seek to explain. However, to use them for the problem at hand, they would have to be implemented in 3D at the meso- or macroscale, which is computationally extremely challenging. In addition, robust approaches are still lacking for the up-scaling of physical rock properties from the micro (i.e., grain) scale to the meso (fracture) or macro (fault, stimulated volume) scale.

Statistical approaches to model the seismicity aim at predicting the total number and frequency-magnitude distribution of future seismicity in a given time window. Based on the Gutenberg-Richter law of frequency-magnitude distribution and the Omori law of aftershock decay, the seismic hazard can be forecasted based on the previously observed seismicity catalog (Ogata, 1988; Reasenberg and Jones, 1989; Woessner et al., 2010). Such an approach was also tested on the Basel induced sequence (Bachmann et al., 2011). It is, in principle, independent of the physics of the triggering mechanism. However, the model is static in the sense that it cannot predict the spatiotemporal variability of the \( b \)-value (as observed in Basel by Bachmann et al., 2012), which plays a major role in hazard assessment. It was shown by Barth et al. (2011) that time-decreasing \( b \)-values are required to explain the observed increase in seismic hazard after shut-in.

Observations such as the latter are a starting point for our study. We develop a hybrid model in which we base the rupture initiation on pore pressure diffusion and Coulomb triggering. The size of the earthquake is modeled with a semiprobabilistic approach: We randomly assign event magnitudes from a given Gutenberg-Richter distribution whose \( b \)-value is tied to differential stress. We can therefore forward-model the spatiotemporal variation of \( b \)-values and event stress drop by way of physical modeling where \( b \)-value and stress drop are tied to differential stress. The result is a hybrid geomechanical model of induced seismicity with probabilistic elements that is able to explain the observed source property variations to the first order.

The main aim of this paper is twofold. First, we seek a possible explanation for the observed spatio-temporal behavior of stress drop and \( b \)-value (Goertz-Allmann et al., 2011; Bachmann, 2012). We can reproduce these variations in principle with a simple,
hydraulically homogeneous and isotropic model. Second, we attempt to use the geomechanical model to predict aspects of the behavior of an induced seismicity cloud. The modeled source property variations in space and time and the ability to include event magnitudes allow us to calculate the probability of exceeding a certain magnitude event and therefore make a prediction of the seismic hazard of an injection operation. This last step requires a calibration of the model to observed data. In our case, we use the observed seismicity of the Basel geothermal stimulation (Häring et al., 2008; Deichmann and Giardini, 2009). Due to the calibration step, the obtained probabilities are strictly valid only for the Basel case. However, the main characteristics of the modeled probability curves are independent of the calibration step and allow us to make some generalized statements. To illustrate the sensitivity to the underlying stress regime, we present example calculations of event probability for a few selected stress scenarios. If calibrated repeatedly using an evolving measured seismicity during a stimulation operation, the presented method may be used in the future for a near-realtime prediction of induced seismicity. In addition to assigning magnitudes and stress drop to each event, our rigorous application of Mohr-Coulomb theory can also model repeating events at the same location. Repeating events are important to properly describe effects close to the injection point.

We first present the details of the new hybrid modeling methodology and the calibration to the Basel data before discussing the features of the resulting seismicity cloud. This provides a geomechanical explanation for the variation of $b$-value and stress drop that are observed in reality and successfully reproduced by our model. We then present the calculation for the hazard estimation. At last, we discuss some principle features of the hazard estimates that are independent of the calibration step and may therefore be applicable in different injection scenarios.

**MODELING METHODOLOGY**

We randomly distribute possible failure points (seeds) in a 3D volume with a constant background stress regime and constant friction coefficient and cohesion. At each seed point, we randomly assign values for the minimum and maximum principal stress, $\sigma_2$ and $\sigma_1$, assuming a Gaussian perturbation to a given background stress regime. Focal mechanisms of the largest events in Basel suggest mostly strike-slip motion (Deichmann and Giardini, 2009). Therefore, in Basel, $\sigma_1$ and $\sigma_2$ are both horizontally oriented and $\sigma_2$ would be the vertical stress. However, for our model, the orientation of the background stress regime could be any of normal, thrust, or strike regime. Focal mechanisms of the largest events in Basel suggest that structure tends to be spatially correlated, with, for example, an exponential or von Karman correlation function (e.g., Frankel and Clayton, 1986; Holliger and Levander, 1992). However, it was shown in depth by Rothert (2004) that such a spatial correlation of seed points (the criticality criterion in his case) has little to no influence on the triggered event cloud. Therefore, we decided not to introduce a spatial correlation.

In an injection operation, the stress field is mainly modified by the injection pressure. Increasing the pore pressure $p_p$ in the medium causes a reduction of the normal stress $\sigma_n$ to an effective stress. If the stress is near the critical state (Mohr circle close to the assumed stress regime and perturbations. These cases are based on the Basel geothermal example with numbers taken from Häring et al. (2008) who investigated the depth-dependent stress regime in the Basel-1 borehole. At Basel, injection occurred at a depth of 4.5 km within the Granite. The magnitudes of the principal stresses and their perturbations are also consistent with stress magnitudes modeled from borehole observations by Sikaneta and Evans (2012) and a focal mechanism analysis by Terakawa et al. (2012). Because we require the medium to be in a stable equilibrium before the pore-pressure perturbation, the upper bound of $\sigma_1$ is effectively linked to the coefficient of friction. If we chose a smaller coefficient of friction, all stable systems (Mohr circle just below the failure criterion) will have smaller average values of $\sigma_1$. Note that the initial seed points are randomly distributed in the medium without spatial correlation. Many observations of the statistical properties of elastic parameters in the upper crust suggest that structure tends to be spatially correlated with, for example, an exponential or von Karman correlation function (e.g., Frankel and Clayton, 1986; Holliger and Levander, 1992). However, it was shown in depth by Rothert (2004) that such a spatial correlation of seed points (the criticality criterion in his case) has little to no influence on the triggered event cloud. Therefore, we decided not to introduce a spatial correlation.

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![Figure 1](image-url)
Coulomb envelope), the reduction of the normal stress may cause the shear stress $\tau_s$ to exceed the Coulomb failure envelope and hence trigger an event (Figure 1b)

$$\tau_s = \mu(\sigma_n - p) + c,$$

where $\mu$ denotes the coefficient of friction and $c$ the cohesion. Fault planes are assumed to be always optimally oriented; i.e., at half the angle prescribed by the point at which the Mohr circle is tangential to the failure criterion. If $p$ continues to increase, the failure envelope may be reached again at a later point in time due to the previous event stress drop, thus triggering a repeating earthquake. We model the spatiotemporal evolution of the effective stress based on a linear diffusion model in a hydraulically homogeneous and isotropic medium with a linearly increasing pressure source (Dinske et al., 2010),

$$p(t) = 4\pi Da_0(p_0 + p_1t), \quad 0 < t < t_0.$$

For this case, Dinske et al. (2010) formulate an analytical solution to the diffusion equation (Wang, 2000) that we will use in the following for our modeling. Values for the hydraulic diffusivity $D$, the effective source radius $a_0$, the starting pressure $p_0$, the wellhead pressure gradient $p_1$, and the injection time $t_0$ are listed in Table 1, and are again chosen to closely follow the Basel case study. With the listed values, equation 2 denotes a simplified approximation to the actual wellhead pressure in Basel. We assume an injection point in the center of our volume. The assumption of a point source is corroborated by the shape and evolution of the observed microseismic cloud in Basel which originates from a single point just below the casing shoe. An event is recorded once the Mohr circle has reached the failure envelope.

Note that the number of seed points can be adjusted to modify the number of induced events. We chose the number of seeds (30,000) such that the modeled seismicity’s $a$-value, describing the overall event productivity of the volume (Gutenberg and Richter, 1944) is similar to the $a$-value measured from the Basel event cloud (4.3). The choice of seed points is bounded by considerations detailed in Appendix A. We do not define a minimum and maximum triggering pore pressure (criticality). Theoretically, a stable stress state can be infinitesimally close to the failure envelope, and hence the minimum pore pressure required to trigger an event can be infinitesimally small. In practice, stable systems are systems with a minimum triggering pressure larger than the average background pressure fluctuations imposed by, e.g., tidal variations, ambient seismic noise, etc. Choosing $a_0 = 70$ m and 30,000 seeds results in a triggering front (envelope to all events) that roughly follows the 2000 Pa pressure contour (Figure 2). 2000 Pa is the maximum tidal-induced pressure variation measured from borehole water level gauges in Basel (K. Evans, personal communication, 2011), and therefore a viable assumption for the minimum triggering pressure.

With the analytical solution used in this study, we misuse the factor $a_0$ as a free parameter to fit realistic pressure values near the triggering front. This comes at the expense of incorrect pressure values very close to the injection point (values beyond the 30 MPa contour in Figure 2). More realistic pressure estimates across the whole volume might be achieved by including time-varying permeability enhancement. However, the unrealistically high values very close to the borehole have no impact onto our triggered seismicity cloud and can therefore be ignored. In general, it should be noted that the absolute number of events triggered in the modeling is governed by four parameters: (1) the magnitude of the pressure distribution (controlled by the factor $a_0$), (2) the average differential stress (controlled by our assumption of the background stress field), (3) the friction coefficient, and (4) the seed density. There exist many possible combinations of these four parameters that result in very similar seismicity patterns and $a$-values. The large effective source radius $a_0$ used in our model compared to Dinske et al. (2010) trades off with the number of seed points chosen in the model. The smaller the source radius, the more seed points are required to induce an event cloud with a similar $a$-value. We seek to constrain the realistic parameter range by reasonable a priori assumptions. However, the actual choice of these four parameters has little influence on the conclusion as long as the overall behavior of the seismicity cloud is matched.

Schorlemmer et al. (2005) infer that the $b$-value acts as a stress meter that depends inversely on differential stress $\sigma_d$. This inverse relationship has also been suggested by laboratory studies (Amitrano, 2003). To model the size of each induced event, we link the $b$-value to $\sigma_d$ in an inversely proportional relationship, and then randomly draw a magnitude from a Gutenberg-Richter relation with

### Table 1. Mean medium parameters.

<table>
<thead>
<tr>
<th>Model</th>
<th>Avg. crust</th>
<th>Avg. crust</th>
<th>Weak crust</th>
<th>Strong crust</th>
</tr>
</thead>
<tbody>
<tr>
<td>depth</td>
<td>4.5 km</td>
<td>2.5 km</td>
<td>4.5 km</td>
<td>4.5 km</td>
</tr>
<tr>
<td>$\sigma_3$ (MPa)</td>
<td>75</td>
<td>42</td>
<td>75</td>
<td>75</td>
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<tr>
<td>$\sigma_1$ (MPa)</td>
<td>185</td>
<td>105</td>
<td>147</td>
<td>232</td>
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<tr>
<td>max $\sigma_1$ (MPa)</td>
<td>232</td>
<td>129</td>
<td>232</td>
<td>232</td>
</tr>
<tr>
<td>std. dev. ($\sigma_1$, $\sigma_3$)</td>
<td>10%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_h$ (MPa)</td>
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<td>25</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>$\mu$</td>
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<td>0.85</td>
<td>0.6</td>
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</tr>
<tr>
<td>No. of seeds</td>
<td>30,000</td>
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<td></td>
<td></td>
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<tr>
<td>Cohesion (MPa)</td>
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<td>$a_0$ (m)</td>
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<tr>
<td>$p_1$ (Pa/s)</td>
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<tr>
<td>$t_0$ (s)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$D$ (m$^2$/s)</td>
<td>0.05</td>
<td></td>
<td></td>
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</table>

![Figure 2. Time-distance plot of the simulated induced seismicity. The color denotes the number of repeats of an event. Note that the colorscale is clipped at five repeats. The dashed line shows the shut-in time. The solid contour lines show isobars in MPa.](image-url)
the corresponding $b$-value. It is this inverse relationship that provides the coupling between the physical modeling and the probabilistic magnitude assignment in our model. We seek to base this coupling term on empirical observations to the extent possible. For this purpose, we use the following working hypothesis: (1) the long-term average tectonic $b$-value is one or lower, and (2) the relation between differential stress and $b$-value is actually linear. The latter hypothesis is suggested by the laboratory results of Amitrano (2003), whereas the former assumption results from the observation of $b$-values approaching one for the postinjection seismicity at large distances and long after the injection (Bachmann et al., 2012), as well as mean $b$-values below one observed by Schorlemmer et al. (2004, 2005), and Gulia and Wiemer (2010). First, we choose $\sigma_d = 136$ MPa as the upper differential stress limit, which corresponds to the mean differential stress including respective standard deviations of $\sigma_1$ and $\sigma_3$ for the average crust at 4.5 km depth (see Table 1). We fix the lower $b$-value limit ($b_{\text{max}}$) to one at $\sigma_d = 136$ MPa, but we loop over possible upper $b$-value limits ($b_{\text{max}}$) at the hypothetical value of $\sigma_d = 0$ (inset of Figure 3) and compute the L1 misfit between the mean $b$-value of the observed data and the mean $b$-value from 50 model runs, each with a different realization of random seeds and stresses. The best fit between the Basel data and the modeled seismicity is observed at the minimum of the quadratic least-squares fit ($0.058x^2 - 0.46x + 0.99$), which corresponds to $b_{\text{max}} = 4 \pm 0.24$ (the uncertainty is estimated using Viegas et al. (2010)). We therefore use a linear scaling relation of $b = -0.022\sigma_d + 4$. The resulting relation between $\sigma_d$ and $b$-value is suitable for the Basel data. Additional data points for coincident stress magnitude and seismicity measurements will be required to investigate whether this relation might be varying between locations or stress regimes.

In the next step, we calibrate the modeled seismicity to the overall number of events observed in the Basel data. This calibration step is necessary for the calculation of realistic $a$-values and hence absolute values of forecasted magnitude probabilities. Instead of modifying the seed density, which also controls the overall number of induced events but is computationally very expensive, we estimate a constant scaling factor $s$ from 100 model runs with $s = \langle \sum_{i=1}^{100} N_{\text{observed}}/N_{\text{model}} \rangle / 100$, where $N$ is the overall number of induced events. We obtain $s = 0.92$, and therefore randomly pick 92% of the modeled seismicity of each run for further analysis.

Event stress drop $\Delta \sigma$ is also assumed to be proportional to $\sigma_d$. After an event is induced due to the increase in $p$, $\sigma_1$ is reduced by $10\% \pm 5\%$ of $\sigma_d$. We define the difference in shear stress $\tau_i$ before and after the reduction of $\sigma_1$ to be the stress drop (Figure 1b). Due to the stress drop, the point in the medium becomes stable again and a further increase in $p$ at a later time may repeat the entire process at this point (Figure 1b). Note that the scaling between $\Delta \sigma$ and $\sigma_d$ is arbitrarily defined. A higher percentage would lead to higher absolute $\Delta \sigma$ values and a larger standard deviation to a higher scatter in individual $\Delta \sigma$. The scaling factor used results in a mean unsmoothed $\Delta \sigma$ of 2.56 MPa and a variation over two orders of magnitudes of individual $\Delta \sigma$ values. This is similar to stress-drop estimates obtained for the Basel induced seismicity (Goertz-Allmann et al., 2011).

The result is a forward-modeled seismicity cloud with location, origin time, stress drop, and magnitude assigned to each event location, and calibrated to match the overall number of events observed in Basel. About 1000 events are induced with about 50% of these events being repeaters (seed points rupturing more than once, Figure 2). This is consistent with a multiplet analysis for the Basel microseismic events, which grouped 52% of all events into multiplets (Kummerow et al., 2011). Most repeaters in our model occur close to the injection point, where pore pressures are higher, and the number of repeats decreases with overall pore pressure perturbations, and hence with distance.

### DISTANCE DEPENDENCE OF SOURCE PARAMETERS

Despite restriction to a hydraulically homogeneous and isotropic medium, our model is able to explain the two seismological observations of reduced stress drops (Goertz-Allmann et al., 2011) and increased $b$-values (Bachmann, 2012; Bachmann et al., 2012) near the injection point where pore-pressure perturbations are highest. It also realistically simulates the occurrence of repeating event clusters, consistent with observations (Kummerow et al., 2011). Coloring $b$-value and stress drop (Figure 4a and 4b) for one model run shows the distance-dependence of both compared to the observed Basel seismicity (Figure 4c). To enable a fair comparison, we estimate the $b$-value from the synthetic seismicity cloud in the same manner than for the real data: We compute the $b$-value from the event magnitudes over a fixed number of 100 surrounding events to evaluate the lateral variation following Bachmann et al. (2012). Stress drop is spatially smoothed using a median filter over the closest 20 events. The smoothing of $\Delta \sigma$ is needed to investigate any

![Figure 3. Data-driven definition of the relation between $\sigma_d$ and $b$-value. Modulus of the overall $b$-value difference between the Basel data and the model versus possible upper $b$-value limits ($b_{\text{max}}$). The circles show the mean value from 50 model runs with standard deviation. The solid line shows a quadratic fit with its minimum determined at $b_{\text{max}} = 4$ (dashed line). The respective best fitting scaling relation is shown. The inset shows a sketch of the underlying $b$-value versus differential stress $\sigma_d$ scaling relations using different upper $b$-value limits. The lower $b$-value limit is fixed to one at $\sigma_d = 136$.](image-url)
spatial variability and some spatial smoothing has also been applied to the observed Basel data (Goertz-Allmann et al., 2011). We notice that the variance of the $b$-values and especially of the stress drops is considerable, even though this is synthetic data and we based our modeling originally on a comparatively moderate variance of 10% for the principal stresses. To analyze the properties of the synthetic cloud in a statistically robust way, we run 100 model runs, each with a different realization of the random seeds and stresses. The result of the spatial $b$-value and stress drop variations is summarized in Figure 5. Although Figure 5a and 5b shows a 3D display and a cross-sectional view for one model run, Figure 5c shows a grid-stack of the respective $b$-value (top) and stress drop (bottom) over 100 model runs. Mean $b$-value and mean stress drop show a stable distance dependence from the injection point with high $b$-value and low stress drop at the close distance. Figure 5d shows a corresponding plot for a constant $b$-value of 1.48 and a constant stress drop of 2.5 ± 1.25 MPa, without any dependence on differential stress. In this case, lateral changes in $b$-value or stress drop are random and no distance dependence is observed.

If we plot the source parameters versus distance for the case of stress-coupled $b$-values (Figure 6), we observe that the distance dependence is strongest up to 200–300 m and decreases at further distances. This is especially visible if we compute the mean of the mean values of each distance bin from 100 model runs (Figure 6c and 6d). The error bars in Figure 6a and 6b show the standard error of the mean values and have been computed using bootstrap resampling over 1000 realizations. The error bars in Figure 6c and 6d show the standard deviation.

The higher the pore-pressure perturbation, the less critical stress states still trigger an event and hence the lower the differential stress can be before triggering an event (Figure 7). The inset in Figure 7a shows the decrease of differential stress with increasing pore pressure. This dependence results in lower differential stress close to the injection point and an increase of differential stress at further distances (Figure 7b). Therefore, less critical stress states result in lower stress drops and higher $b$-values, if both are linked to differential stress. We expect the lowest $\Delta \sigma$ and highest $b$-value close to the injection point where the pore pressure is the highest. In addition to the effect described above, the distance dependence of $b$-value and stress drop is strengthened by repeating events. Each repeat at one location results in a smaller differential stress than its predecessor due to the previous event stress drop. This leads to increasingly higher $b$-value and decreasingly lower stress drop for each repeater. Because the number of repeaters increases close to the injection point (Figure 2), the distance dependence of stress drop and $b$-value is emphasized.

**BREAK IN SELF-SIMILARITY OF SEISMICITY**

Many source parameter studies indicate that $\Delta \sigma$ is scale invariant for natural seismicity (e.g., Abercrombie, 1995; Allmann and Shearer, 2009), as well as for induced seismicity (e.g., Kwantek et al., 2010; Goertz-Allmann et al., 2011). However, there are also various studies that found an increase of stress drop with magnitude (e.g., Mayeda and Walter, 1996), and this topic remains a controversial question with important implications for seismic hazard analysis.

Our model implicitly assumes a connection between $b$-value and stress drop because both parameters are linked to differential stress. We therefore indirectly input a nonself-similar stress drop scaling with magnitude in our model. Figure 8 shows the relation between stress drop and magnitude for different assumptions. No dependence of stress drop and magnitude is observed if the stress drop is linked to differential stress but includes a 5% scatter (Figure 8a), and we cannot distinguish the result from a constant stress drop model (Figure 8c). A slight increase of stress drop with magnitude is observed for the raw stress drop values only if the additional 5% scatter of the stress drop is omitted in the modeling (top of Figure 8b). This can be better seen if we directly compare the mean values (Figure 8d). If a spatial smoothing is applied to the individual stress drops (bottom row of Figure 8), the expected dependence with magnitude becomes indiscernible. This shows that even synthetic stress drop scaling is difficult to resolve. It appears to be an extremely challenging task to extract empirically sound scaling relations from real data, where we have to deal with considerable additional scatter due to noise and measurement errors. Furthermore, our result suggests that a spatial smoothing of parameters should not be applied if scaling relations are investigated.
TIME AND LOCATION OF LARGE MAGNITUDE EVENTS

Several conditions must be met for significant (damaging) earthquakes to occur. There must be a fault large enough to allow significant slip, there must be stress present to cause the slip along the fault and this stress must exceed the strength of the fault. It is not clear today what is the upper possible magnitude limit ($M_{\text{max}}$) for induced events in a comparatively shallow reservoir setting (<5 km depth). However, this is an important input parameter to probabilistic seismic hazard assessment.

Using our model, we investigate the probability of an event exceeding a certain magnitude with respect to injection time and distance from the injection point (Figure 9). We evaluate the events for $a$- and $b$-values within specific time and distance bins. For each bin, the probability $P$ of a certain magnitude $M$ event is defined as (Wiemer, 2000),

$$P = 1 - e^{-(a-Mb)}.$$  \hspace{1cm} (3)

We choose time bins of $10^5$ s (1.16 days), moving at $10^4$ s (3.84 h) intervals, and distance bins of 100 m, moving across distance in 10 m increments. To obtain more stable results, we analyze the induced seismicity from 100 model runs and stack individual probabilities. This allows us to compute a standard deviation to each distance bins of 100 m, moving across distance in 10 m increments. To obtain more stable results, we analyze the induced seismicity from 100 model runs and stack individual probabilities. This allows us to compute a standard deviation to each

Note that the simple, hydraulically homogeneous, and isotropic model is sufficient to explain the principle phenomenology. The accuracy of the prediction could certainly be improved by taking into account more realistic hydraulic models. In Basel, for example, anisotropy of hydraulic diffusivity is probably significant judging from the shape of the event cloud. This would have an effect on

Figure 5. Spatial $b$-value (upper panels) and smoothed stress drop (lower panels) distributions of induced events in (a) 3D view for one model run, (b) cross section for one model run, (c) grid-stack of all cross sections over 100 model runs, and (d) grid-stack of all cross sections over 100 model runs using a constant input $b$-value and stress drop as described in the text. The cross sections include all events within 100 m to a plane through the injection point.
lateral variation of seismic hazard. On the other hand, the analytic solution available for the simple homogeneous isotropic case enables fast computation. A more complex hydraulic model would require a numerical modeling approach which may be incompatible with the requirement of a real-time implementation in an advanced traffic light system.

**DEPENDENCE ON DEPTH AND CRUSTAL STRENGTH**

To test a more general applicability of the simple geomechanical model for the purpose of forward-modeling expected seismic responses to fluid injections, we now investigate the influence of some of the modeling parameters onto the resulting seismicity cloud and source parameters. First, we calculate one model with an injection at a shallower depth of only 2.5 km, with correspondingly smaller overall stress magnitudes (Häring et al., 2008) and with only half the pore pressure. In a second step, we vary the strength of the crust by adjusting $\sigma_1$ and the coefficient of friction $\mu$. Input parameters of the three additional models are shown in Table 1.

We keep the scaling relation between differential stress and $b$-value, respectively stress drop, exactly the same as before, even though this dependence was obtained heuristically on the basis of the Basel observations. Varying differential stresses between the three different scenarios therefore lead to varying absolute values of average stress drop and $b$-value. Our main interest is the relative variation of the source parameters and probabilities with time and distance. These relative variations can be extracted reliably from the modeling runs, and some more general postulates about induced seismicity can be derived despite variations in absolute values. It is unclear at this point whether any scaling between differential stress and $b$-value or stress drop would be varying in different geologic situations. However, for the time being we consider it...
a realistic assumption because it explains the observation at Basel quite well.

At shallow depth and for a weak crust, we observe overall much higher $b$-values above two (stars and diamonds, respectively, Figure 10) compared to the original model at 4.5 km depth (circles), for which we obtain a good fit to the observed Basel seismicity (squares). The strong crust model results in a much lower $b$-value (inverted triangles). The stronger the distance dependence of the $b$-value, the worse the fit of the data to a constant average $b$-value. This is particularly evident for the strong crust model (inverted

Figure 8. Stress drop versus magnitude for (a) the original model where stress drop is linked to differential stress and a 5% scatter is included, and (b) the same as (a) but without the scatter. (c) The constant stress drop of 2.5 ± 1.25 MPa and (d) the comparison of the mean values of (a) to (c). Upper row shows raw stress drops and bottom row spatially smoothed stress drops. The bold symbols show the mean value within 0.2 magnitude bins with respective standard error from bootstrap resampling using 1000 realizations.

Figure 9. Probability of an event exceeding a magnitude (a) M3, (b) M4, and (c) M5 to occur at a certain time (top row) and distance from the injection point (bottom row). Error bars show the standard deviation computed from 100 model runs. The dashed line marks the shut-in time in time, and the location of the largest observed Basel event in distance. The different colors denote the model where the $b$-value is linked to differential stress (white), and the model with a constant input $b$-value (gray).
triangles). The distance dependence of the $b$-value is greatly reduced for the shallow and weak crust models due to the reduced range of differential stresses (Figure 11). Interestingly, the stress drop does not show a strong difference in the distance dependence between the different models.

In the next step, we impose a depth-dependent gradient onto the background stress field for the average crust case and an injection source at 4.5 km depth. Following Häring et al. (2008), the background stress field changes as follows

$$p_b(z) = 10 \frac{\text{MPa}}{\text{km}} z$$

$$\sigma_1(z) = 42 \frac{\text{MPa}}{\text{km}} z \pm 10\%$$

$$\sigma_3(z) = 17 \frac{\text{MPa}}{\text{km}} z \pm 10\%.$$  

Because we leave the relation between differential stress and $b$-value the same, we would expect to observe a depth dependence of the $b$-value on top of the radial dependence from the injection point. Figure 12 shows that this is indeed observed. However, the depth dependence is a secondary effect to the radial dependence and therefore difficult to observe in practice. Revealing the mean depth dependence in the synthetic data requires correcting each event first by the radial dependence of $b$-value and stress drop as shown in Figure 6c and 6d, respectively. The result is a variation of $b$-values from 1.4 near the top of the seismicity cloud to 1.2 for the deepest events (diamonds in Figure 12a). Similar to the $b$-value, a weak depth dependence is also imprinted onto the stress drops (Figure 12b). Figure 12c and 12d shows the depth variation of $b$-value and stress drop of the observed Basel data. As expected, the large scatter in the data does not allow us to observe such rather subtle depth variations in reality. Although at least the mean $b$-values per depth bin (white squares in Figure 12c) are not incompatible with a depth variation, the scatter is too large to reliably resolve a depth dependence with statistical significance. The observed stress drops show no indication of a depth dependence.

In summary, our results suggest that the probability of large events ($M \geq 4$) is reduced if we either stimulate less deeply, or drill...
into softer formations such as sediments, instead of granite (Figure 13a and 13b). For the strong crust model, the probability of a $M \geq 4$ event is overall higher compared to the average crust model at 4.5 km depth (Figure 13c).

However, because we do not know whether our scaling between $b$-value and $\sigma_d$ is universally valid, or only valid for Basel, we cannot interpret the absolute probability levels anywhere other than Basel where we derived our scaling relation. Nevertheless, this does not prevent us from comparing the relative probabilities within one scenario before or after shut-in or with distance. The common feature of most modeled scenarios is the observation of the highest probability of a large magnitude event after well shut-in and at larger distances to the injection point. To show this more quantitatively, we calculate normalized cumulative probabilities in time and radial distance to the injection point (Figure 14) for the different model set-ups. For each case, we compare the cumulative probability ratios before and after the shut-in time, as well as closer and further away than 300 m distance to the injection point. Individual values are listed in Table 2. Note that the 300 m distance level is arbitrarily chosen to separate the induced seismicity into two equal groups. For example, we find that our model predicts a 61.5% probability for a $M \geq 4$ event after the shut-in time, and a 53.2% probability for an event beyond 300 m if $b$-value is coupled to differential stress. These probabilities are higher than their respective counterparts before the shut-in and closer than 300 m,
respectively. A constant $b$-value always predicts higher probabilities before the shut-in time and closer than 300 m to the injection point compared to the model where $b$-value is linearly linked to differential stress. Whereas the distance dependence is reduced for the shallow crust model, we also find that the weak and strong crust models predict higher probabilities of a $M \geq 4$ event after shut-in and at larger distances. Two important consequences arise from this observation: (1) it is insufficient to shut-in an injection operation upon observation of a threshold magnitude event (so-called traffic-light system), because the largest magnitude probability is yet to come after the shut-in, irrespective of the applied threshold magnitude. (2) increased probabilities for larger magnitude events at larger distances from the well means that a larger area around the injection well is potentially affected by increased seismic hazard. Although risk mitigation is always possible by limiting the injected volume, the decision process to alter the operation for risk mitigation has to start at a much earlier time and needs to take into account the time variability of the $b$-value.

Figure 14. Normalized cumulative probability of an event exceeding a certain magnitude versus time (a and c) and distance to the injection point (b and d). (a and b) Comparison of an average crust model at 4.5 km depth for varying $b$-value (black curves) and constant $b$-value (gray curves). (c and d) Comparison of an average crust at 4.5 km depth (solid black), shallow crust (dashed gray), weak crust (dotted gray), and strong crust model (solid gray). The vertical lines marks the shut-in time (a and c) or the 300 m distance (b and d).

Table 2. Comparison of normalized cumulative probabilities $p_{\text{cum}}$ of exceeding a certain magnitude event in time and distance to the injection point for various model set-ups.

<table>
<thead>
<tr>
<th>Magnitude $M$</th>
<th>$\geq 3$</th>
<th>$\geq 4$</th>
<th>$\geq 5$</th>
<th>$\geq 3$</th>
<th>$\geq 4$</th>
<th>$\geq 5$</th>
<th>$\geq 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative probability [%]</td>
<td>Varying $b$</td>
<td>Constant $b$</td>
<td>2.5 km</td>
<td>Weak</td>
<td>Strong</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before shut-in</td>
<td>55.3</td>
<td>38.5</td>
<td>23.2</td>
<td>75.5</td>
<td>73</td>
<td>68.4</td>
<td>65.3</td>
</tr>
<tr>
<td>After shut-in</td>
<td>44.7</td>
<td>61.5</td>
<td>76.8</td>
<td>24.5</td>
<td>27</td>
<td>31.6</td>
<td>34.7</td>
</tr>
<tr>
<td>$\leq 300$ m</td>
<td>54.7</td>
<td>46.8</td>
<td>39.4</td>
<td>72.5</td>
<td>70</td>
<td>64.3</td>
<td>60.1</td>
</tr>
<tr>
<td>$\leq 300$ m</td>
<td>45.3</td>
<td>53.2</td>
<td>60.6</td>
<td>27.5</td>
<td>30</td>
<td>35.7</td>
<td>39.9</td>
</tr>
</tbody>
</table>
Note that, for a constant $b$-value, the injected volume is proportional to the total seismic moment (McGarr, 1976), whereas for a varying $b$-value the injected volume is proportional to the overall number of events (Shapiro et al., 2010). We speculate that the correlation between differential stress and pore pressure perturbation (inset in Figure 7a) is the main driver in a hydraulic stimulation setting. This is corroborated not only by observations of Bachmann et al. (2012), but also by $b$-value observations in hydrocarbon reservoir settings (Maxwell et al., 2009; Wessels et al., 2011): High $b$-values when new (tensile) fractures open (previously stable points with corresponding low differential stress) due to high pore pressure close to the injection and low $b$-values when preexisting fractures (with larger differential stress, i.e., higher criticality) are reactivated.

Our study suggests that a priori information about the stresses in the area of interest are important for some initial estimation of the expected seismic hazard. Furthermore, the implementation of a near-realtime probabilistic seismic hazard prediction model seems feasible and can be used to replace previously used traffic light systems. Our model can also be used to investigate the effect on the induced seismicity and the respective seismic hazard using varying stimulation approaches such as a different injection flow rate or a different wellhead pressure.

CONCLUSIONS

We propose a simple geomechanical model for induced seismicity that links differential stress to $b$-value and stress drop. With this model, we can forward-model the induced seismic response to a hydraulic injection in space and time. In addition, we can forward-model source parameters such as stress drop and magnitude of events in a stochastic way. The latter allows us to estimate the seismic hazard associated with an injection operation by calculating the probability of exceeding a certain magnitude event. Our model can explain the observation of increasing stress drops and decreasing $b$-values as a function of radial distance from the injection well. The probability of exceeding a certain magnitude event is larger after well shut-in, and reaches further out from the injection point. The overall procedure can be coupled with incoming seismicity data after well shut-in, and reaches further out from the injection point.

APPENDIX A
CONSTRANTS ON THE SEED DENSITY

We assume that every seed point in our volume represents a disk-shaped fracture of random orientation with a radius that depends on the assigned magnitude. Magnitudes are randomly assigned, but follow a Gutenberg-Richter distribution with a prescribed $b$-value. The higher the seed density (seed points per volume), the more are the fractures intersecting each other, up to the point where intersecting fractures are completely separating individual parts of the rock frame from each other. The fracture porosity at which this is reached is termed critical porosity $\phi_c$ (Nur et al., 1998); it is the point beyond which the fractured material would fall apart. Obviously, we have to choose the seed density such that the fracture porosity stays below this critical value.

Assuming circular ruptures, we can define a moment-dependent fault radius (Eshelby, 1957),

$$r = \left( \frac{7 M_0}{16 \Delta \sigma} \right)^{1/3},$$

with $M_0$ the seismic moment, and $\Delta \sigma$ the stress drop, which we will assume constant for simplicity in this exercise. We know that

$$\log_{10} M_0 \propto 1.5 M_{w},$$

and hence $\log_{10} \Delta \sigma \propto 0.5 M_{w}$, with the $M_w$ distribution following a Gutenberg-Richter statistic above the magnitude of completeness. Consequently, the radius $r$ will follow a similar distribution, and we have to find a meaningful average radius $\bar{r}$ over this distribution.

For disk-shaped fractures, the crack density can be given as (Kachanov, 1992)

$$\rho = \frac{N}{V_0 \bar{r}^3}.$$  \hspace{1cm} (A-2)

where $V_0$ denotes a reference volume, $\bar{r}$ is the average radius of a circular rupture for a given event distribution, and $N$ is the overall number of cracks (seeds) in the volume. To define a porosity to a medium cracked in such a manner, we need to assign an opening width or aperture $d$ to these cracks. For an ellipsoidal shape of the crack, the porosity can then be stated as (Saenger et al., 2004),

$$\phi = \frac{4N}{3V_0 \pi r^2 d}.$$  \hspace{1cm} (A-3)

To define a lower critical porosity limit for such a medium, we consider a simplistic thought experiment with uniformly oriented and regularly distributed fracture sets (Figure A-1). We attempt to fill a cube of side length $n \cdot a$ with a network of nonintersecting fractures with diameter $2r$, spaced a distance $a$ from each other. In this manner, we can fill the cube with 1/2 $n$ cracks in the direction of the two long axes of the crack, and $n$ cracks in the direction of the aperture $d \ll r$, such that to fill the whole cube, it requires $1/4 n^3$ cracks. Assume now that the medium is intersected by three mutually perpendicular fracture networks with a total of $3/4 n^3$ cracks. In order for these fracture networks to fully intersect each other, the crack radius $r$ needs to be such that it describes the circumscribed circle of a square of side length $a$,

$$a = r\sqrt{2}. \hspace{1cm} (A-4)$$
Using equation A-3, we can now state the porosity for this case as

$$\phi_c = \frac{1}{2\sqrt{2}} \pi \alpha,$$  \hspace{1cm} (A-5)

where $\alpha$ denotes the crack aspect ratio $d/r$. We assume that for more complicated situations, e.g., randomly oriented fractures, the critical porosity would be higher. Combining equations A-2 and A-3, we can define an upper bound for the crack density $\rho$, which we equal to the seed density in our modeling,

$$\rho \leq \frac{3}{8\sqrt{2}} \approx 0.27,$$  \hspace{1cm} (A-6)

and with that define an upper bound for the number of seeds in our modeling as

$$N \leq \frac{0.27 V_0}{r^3}.$$  \hspace{1cm} (A-7)

To give an example, for an average rupture radius $r$ of 20 m, which corresponds very roughly to assuming a magnitude of completeness (mode of the magnitude distribution) of one, we could fill our 1 km$^3$ modeling volume with a maximum of 33,125 seeds. Using 30,000 seeds, as we have done in our modeling, fulfills criterion A-6, and stays below this simple critical porosity limit.

REFERENCES


