INTRODUCTION

Although the phenomenon of earthquakes induced by the subsurface injection of fluids has been recognized, and the basic mechanisms understood, for many decades (e.g., Healy et al., 1968), the recent increase in seismicity associated with oil and gas development, including large damaging events (e.g., Ellsworth, 2013; Keranen et al., 2013; Hough, 2014; Rubinstein et al., 2014) makes clear the need to better understand the processes controlling such seismicity and to develop techniques to mitigate the associated seismic hazard.

The relationship of fault stress, fault strength, and fluid pressure at the onset of fault slip in the most basic form is given by the modified Coulomb criterion,

$$\tau = \mu (\sigma - P),$$

(1)

where \(\tau\) and \(\sigma\) are the shear and normal stress, respectively, acting on the fault surface, \(P\) is the pore-fluid pressure, and \(\mu\) is the coefficient of fault friction. The term \((\sigma - P)\) is the effective normal stress (Terzaghi, 1925). From equation (1), a fault can be brought to a critical state through an increase of shear stress \(\tau\), a decrease of the normal stress \(\sigma\), an increase of fluid pressure \(P\), or some combination of the three. Increase of pore-fluid pressure is the most widely cited cause of earthquakes induced by human activities (National Research Council, 2012). Consequently, investigations and models of induced seismicity have tended to focus mainly on spatial changes of fluid pressures (Hsieh and Bredehoeft, 1981; Shapiro and Dinske, 2009).

Although the immediate cause of injection-induced earthquakes is the increase of fluid pressure that brings a fault to a critical stress state, models of the spatial changes of fluid pressure alone are insufficient to either predict or understand the space–time characteristics of induced earthquakes. Comprehensive system-level models that couple physics-based simulations of seismicity with reservoir simulations of fluid pressure changes can provide an experimental capability to investigate topics related to induced seismicity. These include (1) investigation of the system-level interactions controlling the space–time characteristics of induced seismicity; (2) characterization of the relationships among injection parameters, reservoir characteristics, and induced seismicity; (3) development of best-practice protocols for injection projects; (4) site-specific models of injection-induced earthquakes; and (5) probabilistic hazard evaluations of the potential for inducing earthquakes. Previous physics-based simulations include McClure and Horne (2011), who investigated the effects of fluid injection on a 1D fracture. Here, we present a modeling approach for simulating seismicity induced by fluid injection in 3D over multiple earthquake cycles.

INDUCED-SEISMICITY SIMULATIONS

We developed a method to simulate injection-induced seismicity that couples the regional scale, multicycle earthquake simulator, RSQSim, to reservoir models that give changes of effective stresses acting on the modeled faults due to fluid injection. RSQSim is a computationally efficient 3D boundary-element code that incorporates rate–state fault friction to simulate long sequences of earthquakes in interacting fault systems. Although RSQSim (Dieterich and Richards-Dinger, 2010; Richards-Dinger and Dieterich, 2012) was specifically developed for efficient simulations of earthquakes and other fault slip processes in geometrically complex fault systems, our initial investigation reported here focuses on the characteristics of induced earthquakes arising from injection near a single isolated planar fault.

RSQSim is a boundary element code in which faults are represented by arrays of rectangular or triangular elements. Because fully dynamic models of the rupture process are computationally intensive, the great efficiency of this code derives in part from use of a quasi-dynamic approximation of rupture dynamics wherein slip speed during an earthquake is fixed at a constant value consistent with the elastic shear impedance and earthquake stress drop. Slip on the fault elements is governed by a rate- and state-dependent constitutive formulation of friction,

$$\tau = (\sigma - P)\left[\mu_0 + a \ln \left(\frac{\dot{\delta}}{\delta^*}\right) + b \ln \left(\frac{\theta \delta^*}{D_c}\right)\right],$$

(2)

(Dieterich, 1978, 1979; Ruina, 1983; Marone, 1998), in which \(a\) and \(b\) are experimentally determined dimensionless constants (with typical laboratory values of $10^{-2}$), \(\dot{\delta}\) is the instantaneous slip speed, \(\delta^*\) is a reference slip speed, \(\mu_0\) is the steady-state coefficient of friction at the reference slip speed and constant normal stress, \(D_c\) is a characteristic distance over which \(\theta\) evolves, and \(\theta\) is a state variable (with dimensions of time) that evolves according to the aging law (with the variable normal stress modification of Linker and Dieterich, 1992):
maximum distance of running mean filter (so that stresses are correlated over a maximum distance of $\sim 170$ m) and (2) the remnant pattern after an $M_r > 5$ rupture from a previous simulation (Figs. 2 and 5). The effective normal stress on the fault elements (initially set to 100 MPa) is controlled by fluid pressure histories from an external reservoir simulation of fluid injection. The earthquake simulator can accept pore-fluid pressure (and/or poroelastic stressing) histories from arbitrarily sophisticated reservoir models, but all the results presented here use pressure histories generated from the simple analytic Green’s function for pore-fluid pressure due to injection at a point source into a uniform, isotropic half-space immediately below an impermeable layer, based on Wang (2000). Because of the unbounded nature of the medium we use here, the pressure perturbation due to an infinitely long, constant rate injection will approach a steady-state value at large times. In contrast, injection into a bounded compartment will lead to ever-increasing pore-fluid pressures once the pressure front begins to interact with the boundaries of the compartment.

Even with zero tectonic stressing rates and zero pore-fluid pressure perturbation, earthquakes will nucleate spontaneously if the initial shear stress $\tau_0$ is high enough. Specifically, this will occur if the initial shear stress on any fault element is greater than some $\tau_{\text{max}}$, which is offset above the steady-state stress by an amount controlled by the critical element stiffness for instability (Ranjith and Rice, 1999), which in turn is inversely proportional to $D_c$. In these simulations, we use $D_c = 10$ $\mu$m, which results in very large values for the critical stiffness compared with the element stiffness. In this case, the offset is very small and $\tau_{\text{max}}$ is nearly the steady-state friction, giving

$$\tau_{\text{max}} \approx \sigma_0 \left( \mu_0 + (b - a) \ln \left( \frac{\theta_0 \delta_s}{D_c} \right) \right),$$

in which $\sigma_0$ is the initial normal stress and $\theta_0$ is the initial value of the state variable on that element. In the results we show here, we use initial shear stresses strictly less than $\tau_{\text{max}}$ so that any earthquakes in the simulations would not have occurred in the absence of the injection-induced pore-fluid pressure perturbations.

Similarly, if the tectonic stressing rates are zero and the initial shear stresses are everywhere less than a critical value $\tau_{\text{min}}$, then no events will ever nucleate. An analogous argument to that leading to equation (4), but including the effect of the changing effective normal stress on the state variable, gives

$$\tau_{\text{min}} \approx (\sigma_0 - P_{\text{max}}) \left[ \frac{\mu_0 + (b - a) \ln \left( \frac{\theta_0 \delta_s}{D_c} \right)}{b} \right] - \frac{b}{b} \ln \left( \frac{1}{\sigma_0} \right),$$

in which $P_{\text{max}}$ is the largest pore-fluid pressure perturbation at the location of a given fault element.

Figure 1 illustrates the fault model and some features of a typical simulation. The fault is a 3 km $\times$ 5 km planar surface and consists of 37,500 20 m $\times$ 20 m elements. The upper edge of the fault is buried at 3 km depth. The injection point is even with the top edge of the fault and 200 m from the nearest point on the fault. The injection rate is 0.01 m$^3$/s (26.9 $\times$ 10$^3$ m$^3$/month), which is roughly comparable to injection rates associated with the Rocky Mountain Arsenal induced earthquakes ($7.5 \times 10^3$ to $17 \times 10^3$ m$^3$/month; Healy et al., 1968) and Paradox

(a) Fault surface (gray) with hypocenter locations (colored by time since injection began and scaled by magnitude). The injection well is located at 1.5, -0.2, and -3 km in the along-strike, out of plane, and depth directions, respectively. The contours indicate pressure change (in MPa) at the end of the 20-year injection interval. The background gray scale gives the pattern of initial shear stress with a mean $\tau_0 = 50$ MPa. (b) Inset shows pressure histories at three locations indicated by matching colored stars. Distances are measured along strike from closest point on the fault to the well. (c) Magnitude versus time for the simulation shown in (a). Beginning and end of injection period shown by the blue and red dashed lines, respectively.

$\dot{\theta} = 1 - \frac{\dot{\theta}_0}{D_c} - \alpha \frac{d \theta}{d \sigma} \dot{\sigma}$

in which $\alpha$ is an additional empirical constant and overdots indicate time derivatives. A key property of rate-state friction is that nucleation of unstable slip is highly time dependent (Dietrich, 1992), which results in earthquake clustering, foreshocks, and aftershocks that follow the Omori aftershock decay law (Dietrich, 1994).

For the simulations of injection-induced seismicity described below, tectonic stressing rates are set to zero in order to simulate regions that were formerly tectonically active but are now completely or nearly inactive. We construct models with two different patterns of initial shear stress heterogeneity: (1) Gaussian white noise with a standard deviation of 5 MPa (Figs. 3 and 4) or 15 MPa (Figs. 1 and 6) smoothed with a 3 $\times$ 3 running mean filter (so that stresses are correlated over a maximum distance of $\sim 170$ m) and (2) the remnant pattern after an $M_r > 5$ rupture from a previous simulation (Figs. 2 and 5). The effective normal stress on the fault elements (initially set to 100 MPa) is controlled by fluid pressure histories from an external reservoir simulation of fluid injection. The earthquake simulator can accept pore-fluid pressure (and/or poroelastic stressing) histories from arbitrarily sophisticated reservoir models, but all the results presented here use pressure histories generated from the simple analytic Green’s function for pore-fluid pressure due to injection at a point source into a uniform, isotropic half-space immediately below an impermeable layer, based on Wang (2000). Because of the unbounded nature of the medium we use here, the pressure perturbation due to an infinitely long, constant rate injection will approach a steady-state value at large times. In contrast, injection into a bounded compartment will lead to ever-increasing pore-fluid pressures once the pressure front begins to interact with the boundaries of the compartment.

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Valley, Colorado, induced earthquakes ($\sim 3.7 \times 10^3$ to $5.6 \times 10^3$ m$^3$/month; King et al., 2014). The diffusivity, porosity, and compressibility of the medium are $0.005$ m$^2$/s, $0.05$, and $5 \times 10^{-4}$ Pa$^{-1}$, respectively. With this model, the minimum simulated magnitude is $M_w \approx 1$, and the maximum magnitude is $M_w \approx 5$. In this simulation, there were six earthquakes with $M_w > 3$, in addition to the many smaller events. Generally, the magnitudes of the largest events increase with time and the areal extent of induced earthquakes expands. In this simulation, two small earthquakes occurred after injection was halted at 20 years.

**EFFECT OF INITIAL STRESS ON INDUCED SEISMICITY**

The average initial shear stress $\tau_0$ strongly affects the characteristics of induced seismicity. Figure 2 shows results from three simulations with different average initial shear stresses between $\tau_{\text{min}}$ and $\tau_{\text{max}}$. Consistently, in simulations with increasing $\tau_0$:

- the delay between the start of injection and onset of seismicity decreases,
- the magnitude of the first event increases,
- the magnitude of the largest event in the sequence increases,
- the cumulative number and total moment of induced earthquakes increases,
- the distance from the injection point to the most distant earthquake increases, and
- the seismicity following shut-in increases.

Additionally, the fraction of the earthquake moment due to fluid injection decreases as $\tau_0$ increases (compare the red and black curves in Fig. 2a–c). In these zero tectonic stressing rate simulations with $\tau_0 < \tau_{\text{max}}$, no events would have taken place without the pore-fluid pressure perturbations due to the injection, so in some sense the injection is responsible for all of the events. However, it also is true that without some pre-existing shear stress, no shear-failure events would have occurred no matter how much fluid was injected. To quantify the fraction of the total moment to be attributed to fluid injection versus pre-existing shear stress, we define the portion of the earthquake moment that is due to the increase of fluid pressure as

\[ \frac{M_{\text{inj}}}{M_{\text{total}}} \]

where $M_{\text{inj}}$ is the moment released by all events in the simulation, whereas $M_{\text{total}}$ represents the moment release that can be ascribed to the change in pore-fluid pressure (see text). The along-strike rupture lengths (black horizontal lines) of all events are shown, projected onto the surface of the fault, in (j–l). Gray contours in the lower panels show the pore-fluid pressure with time. Blue and red dashed lines indicate the beginning and end of the injection period in all panels, respectively.
\[ M_{0,p} = M_0 \frac{\Delta \tau_p}{\Delta \tau}, \]  
(6)

in which \( M_0 \) is the total earthquake moment, \( \Delta \tau \) is the earthquake stress drop, and \( \Delta \tau_p \) is the average reduction of fault strength due to the increase in fluid pressure on the fault surface, which is

\[ \Delta \tau_p = \mu \Delta P, \]  
(7)

in which \( \mu \) is the nominal coefficient of friction, which we equate to \( \mu_0 \) in equation (2) and \( \Delta P \) is the average fluid pressure change on the rupture surface of a given event.

The fraction of the moment of individual events that is due to increased fluid pressure decreases with increasing event magnitude (Fig. 3a). By our measure, for this particular simulation at magnitude \( M_w \sim 3 \), more than half of the earthquake moment is a consequence of the increase in fluid pressure due to injection, but at \( M_w \geq 5.0 \) as little as 1% of the moment is due to injection. As suggested by Figure 2a–c, the fraction of the total moment released by an entire induced sequence that can be attributed to injection decreases with increasing initial shear stress (Fig. 3b).

**RELATION OF MAXIMUM MAGNITUDE TO INJECTED VOLUME**

If the average initial shear stress is sufficiently high, then once an earthquake rupture initiates, it will propagate over the entire fault surface. In that case, the maximum possible earthquake magnitude is controlled by the fault dimensions. However, over a wide range of subcritical stresses (\( \tau_0 \) well below \( \tau_{\text{max}} \)), induced-earthquake ruptures are able to propagate only over a sufficiently pressured portion of the fault (Fig. 2). In those cases, the maximum limits of earthquake ruptures (and therefore maximum magnitudes) are related to the volume of the crust over which the pressure is increased sufficiently. This volume, in turn, is proportional to the injected volume (at least during the initial injection period before the pressure field approaches its steady state). This relationship can be derived with simple scaling laws.

The scaling of earthquake magnitude \( M_w \) with rupture area \( A \) can be derived from the definition of earthquake moment \( M_0 \), coupled with the assumption of self-similarity (i.e., constant stress drop) and the moment–magnitude relation. The seismic moment is defined by

\[ M_0 = G \bar{d}, \]  
(8)

in which \( G \) is the elastic shear modulus and \( \bar{d} \) is the average slip. Assuming constant stress drop, the average slip will scale with rupture length \( L \), as \( \bar{d} \sim L \), and therefore (because \( A \sim L^2 \)) with \( A \) as \( \bar{d} \sim A^{1/2} \). Thus, the moment will scale with area as \( M_0 \sim A^{3/2} \). Because the moment–magnitude relation (Hanks and Kanamori, 1979) is \( M_w = (2/3) \log_{10}(M_0) - 6 \), the magnitude will scale with area as

\[ M_w \sim \frac{2}{3} \log_{10}(M_0) \sim \frac{2}{3} \log_{10}(A^{3/2}) \sim \log_{10}(A). \]  
(9)

This same \( M_w \sim \log_{10}(A) \) scaling is also observed in empirical scaling studies (e.g., Wells and Coppersmith, 1994).

For a given volume of pressurized crust \( V \), the linear dimension \( L \) of a fault that will fit in this volume will scale as \( L \sim V^{1/3} \), and hence the area of such a fault \( A \) will scale as \( A \sim L^2 \sim V^{2/3} \). Combining this with the above magnitude–area scaling gives a scaling of magnitude with volume of pressurized crust of

\[ M_w \sim \log_{10}(V^{2/3}) \sim \frac{2}{3} \log_{10}(V), \]  
(10)

**Figure 3.** (a) Ratio of seismic moment due to the change in pore-fluid pressure to the total seismic moment release as a function of magnitude for individual events within a single simulation. (b) Moment ratio for the cumulative moment released in an entire simulation as a function of mean initial shear stress in the simulation.
and, as mentioned above, during the initial injection period the volume of pressurized crust will be roughly proportional to the injected volume so that the scaling of magnitude with injected volume will be the same. Using a different line of reasoning, McGarr (2014) also obtains scaling of \( M_w \sim (2/3) \log_{10}(V) \). Our argument depends on the pore-fluid pressure being dominated by 3D bulk diffusion. If the pressure is instead dominated by fracture permeability along the fault(s) on which the induced seismicity occurs, a similar argument would, in the simplest case, give the scaling of injected volume with fault area (times a fixed fault width and porosity) of \( V \sim A \), resulting in \( M_w \sim \log_{10}(V) \).

Both observations from secondary recovery and waste water disposal wells (National Research Council, 2012) and simulation results show a general agreement with the relationship of equation (10) (Fig. 4). Additionally, in the simulations there is a dependence on initial stress. This arises because higher pressures (larger volumes of injected fluids) are needed to bring the fault to failure for lower initial stresses.

**POST-SHUT-IN SEISMICITY**

Continuing seismicity following shut-in is a common characteristic of injection-induced seismicity. For example, elevated levels of seismicity continued for nearly two years following shut-in of wells in the Rocky Mountain Arsenal near Denver, Colorado (Hsieh and Bredehoeft, 1981). Furthermore, injection for reservoir stimulation at an enhanced geothermal field in Basel, Switzerland, was halted after an \( M_L \) 2.6 event occurred, which exceeded the safety threshold for the stimulation (Deichmann and Giardini, 2009). Several hours following shut-in, an \( M_L \) 3.4 event occurred, the largest earthquake of the sequence (Deichmann and Giardini, 2009). This resulted in measures to reduce reservoir pressure by allowing water to flow back out of the reservoir. Although the flow-back procedure reduced the rate of seismicity, microseismicity was still detected two years later (Häring et al., 2008; Deichmann and Giardini, 2009). This phenomenon is quite important because it means that induced earthquakes cannot necessarily be turned off immediately by termination of injection to manage the risk of a possible future damaging event.

There appear to be two possible mechanisms for continuing seismicity following cessation of injection. First, following shut-in, fluid pressures continue to increase, and therefore to trigger events, until the shut-in signal reaches progressively distant points from the injection well. Second, delayed nucleation resulting from the rate–state properties of faults, can result in continuing aftershocks from both previously induced events and the effective stress perturbation.

Figure 5 illustrates a simulation with a \( \tau_0 \) close to \( \tau_{\text{max}} \) that results in many events following shut-in. A few post-shut-in events occur as the pressure continues to increase at points some distance from the injection well. However, most events occur as the pressure is falling, which indicates that delayed nucleation is responsible. Additionally, we find that the post-shut-in events show a power-law decay (Fig. 5b) with slope near \( -1 \), which is characteristic of delayed nucleation and aftershocks. Once this delayed nucleation process is underway, it is largely self-driven and relatively insensitive to the subsequent stressing history (Dieterich, 1992).

**EFFECTS OF WELL SHUT-IN ON SUBSEQUENT SEISMICITY**

To manage risk associated with induced seismicity, the use of traffic-light procedures has been advocated to progressively scale back or halt injection, based on measures of increasing risk from the recorded seismicity (Majer et al., 2012, 2014; National Research Council, 2012). As an initial examination of the possible utility of this approach, we conduct a set of numerical experiments to determine how far in advance of induced earthquakes shut-in must occur to prevent those particular earthquakes. For this test, we use the simulation shown in Figure 1 as the starting point. We choose three moderate \( (M_w \sim 3-4) \) events from that simulation, labeled A, B, and C in Figure 1, that occur at increasingly later times and distances from the well. In these experiments, the simulation of Figure 1 is repeatedly restarted, and injection is halted at various times before each of the chosen impending earthquakes. Results of the shut-in experiments are shown in Figure 6.

In general, the farther the earthquake from the well, the further in advance of the event that shut-in has to occur to prevent the moderate earthquake. In the case of earthquake A (260 m from the well; Fig. 6a), shut-in any time in the last 14 days before the time of occurrence of the original \( M_w \) 3.8 event leads to essentially the same event (nearly equal magnitude and rupture area) as the original \( M_w \) 3.8, with slightly delayed origin times (not shown). There is a sudden transition such that turning off injection more than 14 days in advance of the original \( M_w \) 3.8 is effective in preventing the occurrence of
any event similar to the original $M_w$ 3.8, with the largest post-injection event having $M_w \leq 2.0$. Neither of the other two events undergoes such a sharp discontinuity in behavior with the variation of shut-in time. Event B ($M_w$ 3.2, 680 m from the well) displays a continuous reduction of rupture area and magnitude as the well is shut-in earlier, such that it requires cessation of injection nearly 5 months in advance of the original $M_w$ 3.2 to keep the magnitude of the largest postinjection event below $M_w$ 2.0. Similarly, for event C ($M_w$ 4.1, 690 m from the well), shut-in approximately 5 months before the original event is required to keep the post-shut-down seismicity below $M_w$ 2.0. For all three of these events, the time after shut-in at which the pore-fluid pressure begins to decrease at the eventual hypocenter (5, 39, and 36 days for events A, B, and C, respectively) is much shorter than the length of time before the original event that injection must be halted to significantly reduce the magnitude of the induced event: the aforementioned self-driven nature of the rate-and-state nucleation process can make it more difficult than might be expected to halt the induced seismicity.

**DISCUSSION**

We presented a modeling approach to investigate the seismic response to fluid injection by coupling RSQSim to a reservoir model. RSQSim is an earthquake simulator that accepts highly complex, 3D fault systems and inputs from geomechanical reservoir models. Such simulations provide a means to conduct experiments to understand the processes controlling induced seismicity under a variety of circumstances.

The results presented here indicate that the space–time patterns of simulated injection-induced seismicity are quite sensitive to preinjection fault stresses. For example, with increasing preinjection shear stress, induced events increase in abundance, expand more rapidly, and post-shut-in seismicity is enriched. Additionally, the portion of seismic moment release that can be attributed to changes in pore-fluid pressure decreases as both earthquake magnitudes and the mean value of $\tau_0$ increases. Although the fluid pressure moment may become very small under these circumstances, the earthquakes would nonetheless not have occurred without the perturbation of fluid pressure. Furthermore, for faults with initial stresses below a near-critical value, the maximum magnitude of induced earthquakes increases roughly as $M_w \sim 2/3 \log_{10}(V)$. This scaling is characteristic of bulk 3D fluid diffusion. Fracture diffusion along the fault would yield scaling by $M_w \sim \log_{10}(V)$. Continuing seismicity following shut-in appears to be driven primarily by delayed nucleation and decays by the Omori aftershock law. Preliminary tests of traffic-light procedures, where injection wells are shut-in to prevent future events, indicate that earthquake magnitudes can continue to increase following shut-in due to both the delay time before the shut-in pulse reaches the most distant sites of impending earthquakes and the self-driven nature of the earthquake nucleation process.

The initial work presented uses an extremely simple fault and injection system. But even within this simple framework, much work remains to be done, for example, on the effects of more complex injection histories and the efficacy of producing from, instead of merely shutting in, an injection well for preventing subsequent induced seismicity. Beyond that, more realistically complex fault geometries and reservoir models need to be explored in order to attempt to successfully model specific observed induced-seismicity sequences. If such sequences can, in fact, be modeled, there is some hope that such simulations could be used in a predictive fashion. Insufficient knowledge of the initial state of stress on the faults will likely be the most serious obstacle, but it may be that comparing the early part of the seismic response to results of many simulations run ahead
of time using a suite of initial stress states may provide information on the state of stress and enable forecasting of the subsequent seismicity.

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