Quantifying Anderson’s fault types

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Abstract. Anderson [1905] explained three basic types of faulting (normal, strike-slip, and reverse) in terms of the shape of the causative stress tensor and its orientation relative to the Earth’s surface. Quantitative parameters can be defined which contain information about both shape and orientation [Célèrier, 1995], thereby offering a way to distinguish fault-type domains on plots of regional stress fields and to quantify, for example, the degree of normal-faulting tendencies within strike-slip domains. This paper offers a geometrically motivated generalization of Angelier’s [1979, 1984, 1990] shape parameters $\phi$ and $\psi$ to new quantities named $A_\phi$ and $A_\psi$. In their simple forms, $A_\phi$ varies from 0 to 1 for normal, 1 to 2 for strike-slip, and 2 to 3 for reverse faulting, and $A_\psi$ ranges from 0° to 60°, 60° to 120°, and 120° to 180°, respectively. After scaling, $A_\phi$ and $A_\psi$ agree to within 2% (or 1°), a difference of little practical significance, although $A_\psi$ has smoother analytical properties. A formulation distinguishing horizontal axes as well as the vertical axis is also possible, yielding an $A_\phi$ ranging from -3 to +3 and $A_\psi$ from -180° to +180°. The geometrically motivated derivation in three-dimensional stress space presented here may aid intuition and offers a natural link with traditional ways of plotting yield and failure criteria. Examples are given, based on models of Bird [1996] and Bird and Kong [1994], of the use of Anderson fault parameters $A_\phi$ and $A_\psi$ for visualizing tectonic regimes defined by regional stress fields.

Introduction

Anderson [1905, 1951] postulated a fundamental relation between the three basic fault types and the orientation of the causative stress tensor relative to the Earth’s surface: new faults will be normal, strike-slip, or reverse depending on whether the maximum, intermediate, or minimum compressive principal axis, respectively, is most nearly vertical. Almost every introductory geology and structural geology textbook has figures displaying this relationship. Anderson also pointed out that the Earth’s surface is a free boundary (no shear or normal traction), so that one principal axis of crustal stress tensors will commonly be close to vertical and the other two axes close to horizontal, especially as the free surface is approached. Although not every crustal stress tensor of interest has this property (because of the effects of topography, heterogeneity, and zones of structural weakness), it is approximated commonly enough to make Anderson’s observation a powerful simplification [e.g., Zoback et al., 1989; Zoback, 1992].

Because of the great utility of Anderson’s fault types for understanding and characterizing tectonic regimes, geologists have extended these types to include transitional cases such as “strike-slip with normal” [e.g., Philip, 1987; Guiraud et al., 1989] and have devised ways to quantify these types with the help of tensor shape parameters (summarized by Célèrier [1995, Table 3]). Coblenz and Richardson [1995] and Müller et al. [1997] have shown the additional usefulness of a quantitative parameter for averaging regional stress data and for plotting and visualizing regional stress fields.

Stress field maps often use scaled symbols or line segments to show length and orientation of the principal axes of the stress tensor at selected points [e.g., Richardson et al., 1979; Bird, 1996]. Sometimes stress trajectories are plotted to display the orientation of maximum or minimum horizontal axes [e.g., Hansen and Mount, 1990; Philip, 1987, Figure 29; Bird and Li, 1996]. Such maps, although dense with information, are often difficult for the eye to assimilate. Maps that are contoured or colored using a quantitative measure of tectonic regime can greatly aid visualization [e.g., Müller et al., 1997].

In this paper, I discuss a geometric approach to quantification based on two specific shape parameters, $\phi$ (or $\Phi$) and $\psi$, used by Angelier [1979, 1984, 1990] in methods he devised for inferring regional stress information from fault slip data. Although the new generalized parameters presented here are similar to others that have been previously suggested, in particular to $\theta$ of Armijo et al. [1982], the geometric derivation offers insights that are not so apparent in more analytical approaches. It should be noted that not all stress fields of interest will satisfy the assumption of a near-vertical principal axes and that the presence of preexisting planes of weakness can lead to faulting types different from Anderson’s pure types [e.g., Bott, 1959; Célèrier, 1995].

Generalizing Angelier’s Shape Parameters

Parameter Based on $\phi$

The purpose of a shape parameter is to convey, in a single quantity, information about the relative magnitudes of the three principal axes of the tensor. For example, if $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the magnitudes of maximum, intermediate, and least principal stress, respectively, with compression being positive, then $\phi = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ is one possible measure of shape. This definition essentially compares the magnitude of...
the intermediate axis $\sigma_2$ to the other axes: if the intermediate axis is close in size to the minimum axis, $\phi$ approaches 0; if close in size to the maximum axis, $\phi$ approaches 1; if exactly halfway between, $\phi = 0.5$. Angelier points out that for many purposes, knowledge of the shape alone permits useful inferences even in the absence of complete information about the actual values of $\sigma_1$, $\sigma_2$, and $\sigma_3$.

In the definition of $\phi$, only the relative sizes of the principal stresses come into play; no particular regard is given to which of the three principal values belongs to the most nearly vertical principal axis. As will be shown, if one distinguishes the vertical axis during the derivation, one arrives at a continuously varying generalized shape parameter which contains information about fault type, as well as the shape of the tensor. Célerier [1995] uses the words "intrinsic" and "vertical" to distinguish these two kinds of stress tensor shape parameter.

Angelier's $\phi$ arises naturally from his corresponding "reduction" of a general stress tensor. The reduction is carried out as follows: Starting with a general stress tensor $T$ expressed in diagonal (principal axis) form with $\sigma_1 \geq \sigma_2 \geq \sigma_3$, first subtract $\sigma_3$, the minimum principal stress, from all diagonal elements and then divide the new diagonal elements by the maximum stress difference ($\sigma_1 - \sigma_3$). The end result is that $T$ can be rewritten as $T = k_1 I + k_2 T_\phi$ where $k_1 = \sigma_3$, $I$ is the identity matrix, $k_2 = \sigma_1 - \sigma_3$ and the reduced matrix $T_\phi$ is given by

$$T_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The first term $k_1 I$ is an isotropic tensor and the second term $k_2 T_\phi$ is a nonisotropic tensor. $T_\phi$ contains all the inherent shape information in $T$. Conversely, one can reconstruct a general stress tensor in diagonal form knowing the shape ratio $\phi$, the maximum stress difference $\sigma_1 - \sigma_3$, and the minimum compressive stress $\sigma_3$. Note that Angelier's reduction was carried out without regard to which of the three principal axes is vertical, so that $\phi$ contains no information on that subject.

**Generalized Parameter $A_\phi$**

A generalization of $\phi$ called $A_\phi$ can be devised that includes shape as well as information about the vertical principal stress axis and, thereby, the fault type. To calculate $A_\phi$ for a stress tensor, define a right-handed coordinate system $(\alpha, \beta, \gamma)$ in stress space. Let the $\gamma$ axis coincide with the (most nearly) vertical principal axis, and let the $\alpha$ and $\beta$ axes be the horizontal principal axes (for the moment, without regard to their relative magnitudes).

Angelier's original $\phi$ reduction can be viewed geometrically in Figure 1. His first step, removal of an isotropic component, shifts a general point $P$ at $(\sigma_\alpha, \sigma_\beta, \sigma_\gamma)$ parallel to the line $\sigma_\alpha = \sigma_\beta = \sigma_\gamma$ (which passes through the origin and the point $(1,1,1)$) until it intersects one of the quarter planes that bound the positive octant (point $Q$ in Figure 1, assuming in this example that $\sigma_\gamma$ is the minimum principal stress $\sigma_3$). The shifted point must lie on one of these quarter planes, because the smallest component of the partially reduced tensor, whether it be $\sigma_\alpha$, $\sigma_\beta$, or $\sigma_\gamma$, is now zero. The next step, which corresponds to extracting the multiplier $k_2$, shifts the point along a line passing through point $Q$ and the

![Figure 1](image_url)

Figure 1. Oblique view of the three-dimensional $(\sigma_\alpha, \sigma_\beta, \sigma_\gamma)$ stress axis system in stress space, showing the steps in Angelier's $\phi$ reduction leading to his definition of shape parameter $\phi$. It is assumed that the example stress tensor at point $P$ started with $\sigma_\gamma > \sigma_\alpha > \sigma_\beta$. Note that $\alpha$, $\beta$, and $\gamma$ are used as shorthand for stresses $\sigma_\alpha$, $\sigma_\beta$, and $\sigma_\gamma$. Boldface N, S, and R stand for the three Anderson fault types (normal, strike-slip, and reverse), with the subscript indicating which of the horizontal axes ($\sigma_\alpha$ or $\sigma_\beta$) has the larger value. The small numbers at the corners of the unit squares indicate the values of $A_\phi$ as defined in the text. Point $P$ locates the original unreduced stress tensor, $Q$ shows its location after the first reduction step, and $R$ is its location after the second step.
origin until it intercepts one of the edges of the unit square (point R in Figure 1). It must lie on an edge because the largest component of the fully reduced tensor is 1. The shape parameter \( \phi \) is now equal to the distance of the point R along the edge of the unit square to the nearest axis. Regardless of which of the six edges the point R ends up on, \( \phi \) is always the distance to the nearest axis.

If we select a starting point on the \( \sigma_r \) axis in Figure 1 and define \( A_\phi \) to measure the distance traversed around the edges of the unit squares in the directions shown by the small arrows, negative for clockwise distances and positive for counterclockwise, then \( A_\phi \) will range from -3 to 3. Using \( \alpha, \beta, \gamma \) as shorthand for \( \sigma_\alpha, \sigma_\beta, \sigma_\gamma \), respectively, and \( \sigma_H \) to indicate the maximum compressive horizontal stress, the correspondences between \( A_\phi \) and \( \phi \) are listed in Table 1.

If \( A_\phi \) is to be calculated for stress tensors at two separated points and stress trajectories are not available to reveal by continuity a consistent choice of \( \alpha \) and \( \beta \) axes for the pair of points, then the horizontal axis with the larger principal stress at each point (commonly referred to as \( S_{\text{max}} \)) can be arbitrarily assigned to be the \( \alpha \) axis. With this assignment, every tensor has an \( A_\phi \) value between 0 and 3, and \( A_\phi \) still contains information about the type of faulting to be expected. The relation to \( \phi \) can be written as an equation: If normal, strike-slip, and reverse types are assigned index numbers \( n = 0, 1, \) and 2, respectively (\( n \) equals the number of principal components larger than \( \sigma_\gamma \)), then

\[
A_\phi = (n + 0.5) + (-1)^n (\phi - 0.5)
\]

where \( A_\phi \) ranges continuously from 0 to 1 for normal, 1 to 2 for strike-slip, and 2 to 3 for reverse faults. Essentially, the corners seen in plots of \( \phi \) as it ranges through the spectrum of tectonic deformation types [e.g., Philip, 1987, Figure 1] have been removed by forcing \( A_\phi \) to increase monotonically across the spectrum. This simplified version of \( A_\phi \) can be used to index all the pure and transitional fault types described by Philip [1987], Guiraud et al. [1989], and Céleriér [1995].

There is another geometric interpretation for the generalized parameter \( A_\phi \). If the stress axis system in Figure 1 is viewed looking toward the origin down the line \( \sigma_\alpha = \sigma_\beta = \sigma_\gamma \), it appears in projection as shown in Figure 2. The edges of the unit squares now form a hexagon, and \( A_\phi \) is proportional to the distance traversed along the perimeter of the hexagon. Although \( A_\phi \) has the virtue of preserving information about both the relative sizes of the axes and the faulting domain, the path traced could be smother.

Table 1. \( A_\phi \) and \( \phi \) Compared

<table>
<thead>
<tr>
<th>( A_\phi ) Range</th>
<th>Faulting Type</th>
<th>Axis Size</th>
<th>( \phi ) Value</th>
<th>( \phi_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3 &lt; A_\phi &lt; -2)</td>
<td>reverse</td>
<td>( \beta &gt; \gamma &gt; \alpha )</td>
<td>( -A_\phi - 2 )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>(-2 &lt; A_\phi &lt; -1)</td>
<td>strike-slip</td>
<td>( \beta &gt; \gamma &gt; \alpha )</td>
<td>( A_\phi + 2 )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>(-1 &lt; A_\phi &lt; 0)</td>
<td>normal</td>
<td>( \gamma &gt; \beta &gt; \alpha )</td>
<td>( -A_\phi )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>(0 &lt; A_\phi &lt; 1)</td>
<td>normal</td>
<td>( \gamma &gt; \alpha &gt; \beta )</td>
<td>( A_\phi )</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>(1 &lt; A_\phi &lt; 2)</td>
<td>strike-slip</td>
<td>( \alpha &gt; \gamma &gt; \beta )</td>
<td>( -A_\phi + 2 )</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>(2 &lt; A_\phi &lt; 3)</td>
<td>reverse</td>
<td>( \alpha &gt; \beta &gt; \gamma )</td>
<td>( A_\phi - 2 )</td>
<td>( \alpha )</td>
</tr>
</tbody>
</table>

Where \( \phi \) is Angelier's [1979, 1984, 1990] shape factor and \( \phi_H \) is the maximum horizontal stress axis.
Generalized Parameter $A_w$

A smoother approach is readily suggested by this projected view; why not use polar coordinates in the plane of Figure 3 (tilted cylindrical coordinates in $\sigma_a, \sigma_b, \sigma_y$ space) so that the distance around a unit circle in the projected plane is the measure of shape? This approach turns out to generalize Angelier’s second measure of shape $\psi$ which corresponds to his second method of reduction [e.g., Angelier et al., 1982; Angelier, 1990, Appendix III]. In the first step of this new reduction, the general point $P = (\sigma_a, \sigma_b, \sigma_y)$ is projected parallel to the line $\sigma_a = \sigma_b = \sigma_y$ until it reaches the plane perpendicular to this line that passes through the origin, becoming point Q in Figure 3. The perpendicular plane has equation $\sigma_a + \sigma_b + \sigma_y = 0$. The projection of $P$ to this plane is equivalent to converting a general stress tensor to deviatoric form, which is accomplished by subtracting the mean stress $\bar{\sigma} = (\sigma_a + \sigma_b + \sigma_y)/3$ from the original diagonal elements.

As before, a general stress tensor $T$ in principal axis form can be written as the sum of a symmetric tensor $c_1 I$ and a nonsymmetric tensor $c_2 T_{\psi}$ (which is the deviatoric stress tensor) containing the essential shape information:

$$ T = c_1 I + c_2 T_{\psi} $$

This is accomplished by introducing a new right-handed $(u,v,w)$ coordinate system in $(\sigma_a, \sigma_b, \sigma_y)$ space with the $w$ axis along the line $\sigma_a = \sigma_b = \sigma_y$ and the $u$ and $v$ axes in the $\sigma_a + \sigma_b + \sigma_y = 0$ plane such that the $u$, $w$ and $\sigma_y$ axes are coplanar. This $(u,v,w)$ axis system is used to define a tilted cylindrical coordinate system in which the polar-coordinate angle $\Psi$ is measured counter-clockwise about the $w$ axis starting at the $u$ axis as shown in Figure 3. The polar radius $\sigma_r$ is given by $\sigma_r = [(\sigma_a - \bar{\sigma})^2 + (\sigma_b - \bar{\sigma})^2 + (\sigma_y - \bar{\sigma})^2]^{1/2}$.

Details are given in the appendix. The result is that

$$ c_1 = \bar{\sigma}; \quad c_2 = \frac{\sqrt{6}}{3} \sigma_r $$

$$ \cos \Psi = \frac{3}{\sqrt{6}} (\sigma_y - \bar{\sigma}) / \sigma_r $$

$$ \sin \Psi = \frac{3}{\sqrt{2}} (\sigma_a - \sigma_b) / \sigma_r $$

$$ A_w = \Psi = \text{ATAN2}[\sigma_a - \sigma_b, \sqrt{3}(\sigma_y - \bar{\sigma})] $$

where ATAN2 is the FORTRAN arc-tangent function with range from $-\pi$ to $\pi$ which preserves quadrant information. $A_w$ is identical to Angelier’s shape parameter $\psi$ to within a phase difference of a multiple of $2\pi/3$. It also has tectonic regime-selecting properties similar to $A_\phi$, although $A_w$ runs from $-\pi$ to $\pi$ (or $-180^\circ$ to $+180^\circ$) instead of from $-3$ to $3$ (Figure 4).

The polar radius $\sigma_r$ is also related to two common quantities

$$ \sigma_r = (2J_2)^{1/2} = \sqrt{3} \tau_{\text{oct}} $$

where $J_2$ is the second invariant of the deviatoric stress tensor and $\tau_{\text{oct}}$ is the octahedral shearing stress [e.g., Jaeger, 1969].

Conversely, the original diagonal stress tensor can be recovered from knowledge of the mean stress $\bar{\sigma}$, the second invariant $J_2$, and the parameter $A_w$. As before, in the absence of information about stress trajectories, it is convenient to assign the maximum horizontal stress to the $a$ axis, so that $A_w$ will vary between 0 and $\pi$. Armijo et al. [1982] defined a similar parameter $\theta = \pi/2 - A_w$ based on analytical rather than geometric intuition, which varies between $-\pi/2$ and $+\pi/2$.

An interesting parallel exists between the above geometric derivation of $A_w$ and the transformations used to convert color coordinates in a three-dimensional red-green-blue (RGB) color space to hue-saturation-intensity (HSI) values [e.g., Fortner

Figure 4. A stress space comparison of the deformation categories of Philip [1987] with the values of $A_\phi$ and $A_w$ viewed projected onto the $\pi$ plane defined by $\sigma_a + \sigma_b + \sigma_y = 0$. Identical categories (not indicated) are mirrored on the right-hand side for negative values of $A_\phi$ and $A_w$. 

radial extension

pure normal

normal / ss transition

pure strike-slip

reverse / ss transition

pure reverse

constriction

radial extension

pure normal

normal / ss transition

pure strike-slip

reverse / ss transition

pure reverse

constriction
Comparison of $A_\phi$ and $A_\psi$

It may not appear at first sight that much has been gained in the definition of $A_\psi$ that could justify the additional algebraic complexity. However, measuring shape by the distance around the circumference of a unit circle is a more symmetric and continuous operation than using the distance around the perimeter of a hexagon as was done for $A_\phi$. Consequently, the $A_\psi$ parameter can be expressed as a single analytical expression of the deviatoric stresses, rather than as a collection of separate expressions which must be pieced together as in the definition of $A_\phi$. In practical terms, however, if $A_\psi$ is scaled to run from $-3$ to $3$, then it differs by only $2\%$ at most from $A_\phi$ when calculated for the same tensor. If $A_\phi$ is scaled to degrees, then the maximum angular difference between the two parameters if they are set equal is $\pm 1^\circ$ or less. This reflects the fact that a hexagon is a fair approximation to a circle for some purposes. Because of this similarity, it may be convenient to use either $A_\phi$ or $A_\psi$ for practical estimation of the quantified Andersonian fault type and to refer to either or both generically with $A$, using no subscript. Figure 5 shows the correspondence between some typical symbols used to plot stress tensors and the corresponding values of the $A$ parameters.

Table 2 and Figure 4 summarize the relation between $A$ ($A_\phi$ or $A_\psi$) and the various generalized tectonic regimes. Names used to describe the regimes are taken from Philip [1987]. Note that the transitional domains such as "normal with strike-slip" are intended to identify a tectonic style in which both normal and strike-slip faulting are present rather than to specifically signify oblique slip with components of both normal and strike-slip offset on a single fault plane. Oblique slip can occur in any of the domains depending on the orientation of a preexisting fault plane relative to the stress field [Wallace, 1951; Bott, 1959; Célérier, 1995]. Preexisting planes of weakness probably explain most of the observed cases of mixing of fault-types in the transitional regions. The "pure" states in Table 2 and Figure 4, ignoring the presence of the Earth's surface for the moment, have stress tensors imposing pure shear in the engineering sense, whereas the radial extension, contructive, and transitional states have stress tensors which correspond to simple extension or contraction in the engineering sense, still ignoring the Earth's surface.

Yield Conditions and Failure

The angular polar coordinate $\Psi$ in Figure 3 contains information about the tectonic regime. The radial polar coordinate $\sigma_r$, which equals the distance from $Q$ to $O$, contains information about the magnitude of the deviatoric stresses and is closely related to several classic measures of failure. The projection of stress space onto the plane $\sigma_r + \sigma_\theta + \sigma_\nu = 0$, sometimes referred to as the $\pi$ plane, is commonly used in discussions of yield and failure [e.g., Fung, 1965, p. 142]. Comparing Figure 4 with Figure 34 of Jaeger [1969], we see that the circle corresponds to a yield surface matching the von Mises criterion and the hexagon matches a yield surface in Tresca's criterion.

Table 2. Comparison of Parameters and Deformation Types

<table>
<thead>
<tr>
<th>$A_\phi$</th>
<th>$A_\psi$</th>
<th>$\phi$</th>
<th>$\gamma$</th>
<th>$\theta$</th>
<th>Deformation Types</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$0^\circ$</td>
<td>$+\pi/2$</td>
<td>$\rightarrow$</td>
<td>radial extension</td>
</tr>
<tr>
<td>0.5</td>
<td>$\pi/6$</td>
<td>$1/2$</td>
<td>$-3$</td>
<td>$+\pi/3$</td>
<td>pure normal</td>
</tr>
<tr>
<td>1</td>
<td>$\pi/3$</td>
<td>$0$</td>
<td>$-1$</td>
<td>$+\pi/6$</td>
<td>normal/strike-slip transition</td>
</tr>
<tr>
<td>1.5</td>
<td>$\pi/2$</td>
<td>$1/2$</td>
<td>$0$</td>
<td>$0$</td>
<td>pure strike-slip</td>
</tr>
<tr>
<td>2</td>
<td>$2\pi/3$</td>
<td>$0$</td>
<td>$+1$</td>
<td>$-\pi/6$</td>
<td>strike-slip/reverse transition</td>
</tr>
<tr>
<td>2.5</td>
<td>$5\pi/6$</td>
<td>$1/2$</td>
<td>$+3$</td>
<td>$-\pi/3$</td>
<td>pure reverse</td>
</tr>
<tr>
<td>3</td>
<td>$\pi$</td>
<td>$0$</td>
<td>$+\pi$</td>
<td>$-\pi/2$</td>
<td>constriction</td>
</tr>
</tbody>
</table>

Here $\phi$ is from Angelier [1979, 1984, 1990], $\gamma$ is from Célérier [1995], $\gamma = \sqrt{3} \tan(A_\phi - \pi/2)$, $\theta$ is from Armijio et al. [1982], $\theta = \pi/2 - A_\psi$. Deformation types are from Philip [1987]; types in angle brackets describe ranges.
In the von Mises criterion, yield is assumed to occur when
\[(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 = 2\sigma_0^2\]  \hspace{1cm} (6)
where \(\sigma_0\) is a constant of the material. This can be shown to be equivalent to requiring that [Jaeger, 1969, p. 93] "yield take place when the elastic strain energy of distortion reaches a value characteristic of the material" or that the octahedral shearing stress \(\tau_{oct}\) (which is the shear stress on a plane equally inclined to all of the principal axes) exceeds a threshold value [Nadai, 1950] which in terms of \(\sigma_0\) equals \(\sqrt{2}\sigma_0/3\). Tresca's condition, which produces a hexagonal yield surface, assumes that yield will occur when the maximum shearing stress \(\sigma_{max} = (\sigma_1 - \sigma_3)/2\) at a point exceeds a threshold value [Jaeger, 1969].

Changing coordinate system in stress space as described in the appendix, a general stress point \((\sigma_x, \sigma_y, \sigma_z)\) in rectilinear coordinates becomes \((\sigma_r, \omega, \phi)\) in the related cylindrical coordinate system. It is not difficult to show that
\[\sigma_r = \sqrt{3}\tau_{oct} = [\csc(\mod(\psi, \frac{\pi}{3}) + \frac{\pi}{6})]\sqrt{2}\sigma_{max}\] \hspace{1cm} (7)

For the von Mises criterion (circle), the proximity to failure is exactly related to the distance of a point from the origin in Figure 4. For the Tresca criterion (hexagon) and for the Coulomb-Navier criterion (irregular hexagon), especially for low values of coefficient of friction, the distance from the origin of yield surfaces onto the \(\sigma_x + \sigma_y + \sigma_z = 0\) plane only works for criteria which do not depend on the mean normal stress. This is not true of the Coulomb-Navier failure criterion, commonly applied to explain frictional faulting of rocks. This criterion has a faceted conical shape in stress space [e.g., Sokolovskii, 1965, p. 7, Figure 3; Scott, 1985, Figure 5; Scholz, 1990, Figure 3.1] unless the coefficient of friction equals zero, so that viewed in a slice cut by plane \(\sigma_x + \sigma_y + \sigma_z = \text{const}\), it will appear as the symmetric but irregular hexagon in Figure 4. The size of the hexagon depends on the slice, and the degree of irregularity depends on the coefficient of friction: if the coefficient is zero, the Coulomb-Navier criterion becomes Tresca's criterion, the hexagon becomes regular, and the conical shape becomes a cylinder.

As Jaeger [1969, p. 97] points out, projecting yield surfaces onto the \(\sigma_x + \sigma_y + \sigma_z = 0\) plane only works for criteria which do not depend on the mean normal stress. This is not true of the Coulomb-Navier failure criterion, commonly applied to explain frictional faulting of rocks. This criterion has a faceted conical shape in stress space [e.g., Sokolovskii, 1965, p. 7, Figure 3; Scott, 1985, Figure 5; Scholz, 1990, Figure 3.1] unless the coefficient of friction equals zero, so that viewed in a slice cut by plane \(\sigma_x + \sigma_y + \sigma_z = \text{const}\), it will appear as the symmetric but irregular hexagon in Figure 4. The size of the hexagon depends on the slice, and the degree of irregularity depends on the coefficient of friction: if the coefficient is zero, the Coulomb-Navier criterion becomes Tresca's criterion, the hexagon becomes regular, and the conical shape becomes a cylinder.

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Plate 1. (a) $A_\phi$ and (b) $\sigma_r$ for the model stress field of Alaska shown in Figure 5 of Bird [1996]. These quantities have been interpolated to a 50 km grid using a minimum curvature gridding algorithm. Short lines indicate the direction of maximum horizontal compression axes. Here $\sigma_r$ equals $\sqrt{3}\tau_{\text{max}}$ and is approximately $\sqrt{2}$ times the maximum shearing stress at any point.
Plate 2. (a) $A_{\phi}$ and (b) $\sigma_r$ for the model stress field of California shown in Figure 12 of Bird and Kong [1994]. These quantities have been interpolated to a 5 km grid using a minimum curvature gridding algorithm. Short lines indicate the direction of maximum horizontal compression axes. Dark contours on Plate 2b are $A_{\phi}$ contours transferred from Plate 2a. Here $\sigma_r$ equals $\sqrt{3} \tau_{oc}$ and is approximately $\sqrt{2}$ times the maximum shearing stress at any point.
origin \( \sigma \), is at least an approximate measure of proximity to failure.

**Examples Using \( A_\phi \) and \( A_\psi \)**

**Coseismic Stress Changes From the 1994 Northridge, California, Earthquake**

An example of \( A_\phi \) used to aid in the visualization of a stress field is shown in Figure 6. This example was calculated from the "combined model" of slip on the Northridge fault plane described by Wald et al. [1996]. Their model consists of slip determinations on 196 rectangular patches on a rectangular planar surface 18 km long horizontally and 21 km long downdip, with a dip of 40° toward an azimuth of 212°. In the combined model, slip was inferred from strong motion, teleseismic, and geodetic data obtained during and after the January 17, 1994, Northridge, California, earthquake of \( M_w \) 6.7. Coseismic stress changes were calculated at the surface for an elastic half-space (shear modulus, 30.0 GPa; Poisson ratio, 0.25) using the dislocation code of Okada [1992]. At depth, stress changes would need to be added to the preexisting stress field in order to yield the postseismic state of stress, but we have chosen to assume in this example that the near-surface geologic materials would be incapable of sustaining sizable tectonic stresses for long periods of time, so that preseismic stress levels were small.

The example was inspired by some of the impressive instances of ground failure that occurred during the earthquake [Hecker et al., 1995; Stewart et al., 1996] and the thought that some of the failure, especially tensional in nature, might be the most important factor involved in the ground failure postearthquake geologic studies have revealed that static deformation, thereby suggesting a correlation. Subsequent instances of ground failure that occurred during the earthquake of stress, but we have chosen to assume in this example that

**Neotectonic Stress Models of Alaska and California**

Plate 1 displays \( A_\phi \) and \( \sigma_\phi \) for a calculated neotectonic stress field of Alaska and the Bering Sea [Bird, 1996, Figure 5]. Bird's [1966] calculation was based on a thin plate, finite element model which included variable crust and lithosphere thickness, heat flow, and elevation. The model also incorporated all faults thought to be active. Geologic slip rates on active faults and geodetic data were used to score the success of 46 different models (all less successful than the one pictured) in which shear from the subduction thrust, fault friction within the North American plate, internal friction of lithospheric blocks, and mantle creep strength were varied.

Plate 1a displays fault-type regimes using color. To make this figure, \( A_\phi \) was obtained for the stress tensor calculated by Bird at the center of each element in the model. (See Bird [1996, Figure 1] for the geometry of the elements.) These discrete \( A_\phi \) values were then interpolated and extrapolated to a 50 km regular grid of values using a minimum curvature algorithm. The grid was further smoothed by linear interpolation to a 10 km spacing for plotting.

The short lines in Plate 1 indicate maximum horizontal stress directions. If new faults were to form in this stress field, the strike of reverse faults would be approximately perpendicular to these lines, the strike of strike-slip faults would be at approximately 45° (or less depending on coefficient of friction) to the trend of these lines, and the strike of normal faults would be approximately parallel to these lines in the respective domains.

Plate 1b shows the quantity \( \sigma_\phi \) summed over the thickness of the lithosphere as by Bird [1966, Figure 5]. As for \( A_\phi \) the irregularly spaced values of \( \sigma_\phi \) at the element centers were gridded using a minimum curvature algorithm. Although Plates 1a and 1b contain no information that was not displayed in the single Figure 5 of Bird [1996], the tectonic content seems easier to grasp.

Similar plots are shown in Plate 2 for a model exploring the behavior of California faults [Bird and Kong, 1994, Figure 12]. The goal of this study was to attempt to infer a time-averaged coefficient of friction acting on faults and an apparent activation energy for creep in the lower crust which would yield a model consistent with known geologic slip rates on faults, principal stress directions, and secular rates of geodetic baselines. A major conclusion of the study was that the major faults are weak with very low coefficients of friction needed to produce the best model results. Although Plate 2 requires two plots and many colors to convey essentially the same information as was shown in Bird and Kong's [1994] Figure 12, that information, especially the expected style of faulting, seems more accessible.

**Conclusions**

Using a geometric approach, it is possible to generalize stress tensor shape parameters in a natural way so that they contain quantitative information about Anderson fault type as well as stress tensor shape. Such quantitative parameters can simplify cataloguing stress data (where information is available) by reducing fault type and tensor shape to a single continuous variable. These parameters can also make the tectonic consequences of local and regional stress fields easier to visualize in plots by using contours or coloring to distinguish tectonic regimes. The geometrically motivated derivation in stress space also leads to a natural way to display information about both tectonic regime and proximity to failure on a single plot and offers a framework for simplifying and motivating some stress tensor calculations.

**Appendix: Details of \( A_\phi \) Derivation**

A new rectangular \((u,v,w)\) axis system is introduced in \((\sigma_u,\sigma_v,\sigma_w)\) space with the \( w \) axis along the line \( \sigma_u = \sigma_w = \sigma_w = 0 \) and the \( u \) and \( v \) axes in the \( \sigma_u + \sigma_v + \sigma_w = 0 \) plane such that the \( u, w \), and \( \sigma_w \) axes are coplanar. The conversion between the \((\sigma_u,\sigma_v,\sigma_w)\) coordinate system and the \((u,v,w)\) system or its related cylindrical \((r,\Psi,w)\) system is most readily obtained by examining how unit vectors in the \( u,v,w \) directions can be expressed in terms of unit vectors in the \( \alpha,\beta,\gamma \) directions. For example, the unit vector in the \( w \) direction is given by
\[ \dot{\psi} = (\dot{\alpha} + \dot{\beta} + \dot{\gamma})/\sqrt{3} \]  
(A1)

A little vector algebra yields expressions for unit vectors in the \( u \) and \( v \) directions. These equations then supply the coefficients for the transformation matrix \( M \) that takes coordinates from the \( (\sigma_u, \sigma_v, \sigma_w) \) to the \( (\sigma_u, \sigma_v, \sigma_w) \) coordinate system:

\[
M = \begin{bmatrix}
-1/\sqrt{6} & -1/\sqrt{6} & 2/\sqrt{6} \\
1/\sqrt{2} & -1/\sqrt{2} & 0 \\
1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3}
\end{bmatrix}
\]  
(A2)

so that

\[
\begin{align*}
\sigma_u &= (2\sigma_y - \sigma_x - \sigma_z)/\sqrt{6} = 3(\sigma_y - \bar{\sigma})/\sqrt{6} \\
\sigma_v &= (\sigma_x - \sigma_y)/\sqrt{2} \\
\sigma_w &= (\sigma_x + \sigma_y + \sigma_z)/\sqrt{3} = \sqrt{3}\bar{\sigma}
\end{align*}
\]  
(A3)

where the mean stress is defined to be

\[ \bar{\sigma} = (\sigma_x + \sigma_y + \sigma_z)/3 \]  
(A4)

Conversion to cylindrical system \( (r, \Psi, w) \) is straightforward:

\[
\begin{align*}
\sigma_r^2 &= \sigma_u^2 + \sigma_v^2 = (\sigma_x - \bar{\sigma})^2 + (\sigma_y - \bar{\sigma})^2 + (\sigma_z - \bar{\sigma})^2 \\
\sigma_r \cos \Psi &= \sigma_u \\
\sigma_r \sin \Psi &= \sigma_v
\end{align*}
\]  
(A5)

The first of these equations just restates the fact that \( \sigma_r \) is the distance of point \( Q \), lying in the deviatoric plane, from the origin in Figure 3. In terms of the FORTRAN ATAN2 function, \( \Psi = ATAN2(\sigma_\alpha, \sigma_\beta) \), and if the stresses \( \sigma_\alpha, \sigma_\beta, \sigma_\gamma \) are deviatoric, this simplifies to

\[ \Psi = ATAN2(\sigma_\alpha - \sigma_\beta, \sqrt{3}\sigma_\gamma) \]  
(A6)

By virtue of the following equations, obtained from (A3)-(A5),

\[
\begin{align*}
(\sigma_\alpha - \bar{\sigma})/c_2 &= \frac{1}{2} \cos \Psi + \frac{\sqrt{3}}{2} \sin \Psi = \cos(\Psi + \pi/3) \\
(\sigma_\beta - \bar{\sigma})/c_2 &= -\frac{1}{2} \cos \Psi - \frac{\sqrt{3}}{2} \sin \Psi = \cos(\Psi + 2\pi/3) \\
(\sigma_\gamma - \bar{\sigma})/c_2 &= \cos \Psi; \\
(\sigma_\alpha - \sigma_\beta)/c_2 &= \sqrt{3} \sin \Psi
\end{align*}
\]  
(A7)

equation (4) in the text becomes

\[
\begin{bmatrix}
\sigma_\alpha & 0 & 0 \\
0 & \sigma_\beta & 0 \\
0 & 0 & \sigma_\gamma
\end{bmatrix} =
\begin{bmatrix}
\cos(\Psi + \pi/3) & 0 & 0 \\
0 & \cos(\Psi + 2\pi/3) & 0 \\
0 & 0 & \cos \Psi
\end{bmatrix}
\]  
(A8)

where

\[ c_1 = \sqrt{3} \]  
(A9)

\[ c_2 = (\sqrt{6}/3) \bar{\sigma} \]

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