Quantitative analysis of rock stress heterogeneity: Implications for the seismogenesis of fluid-injection-induced seismicity

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**ABSTRACT**

We compared elastic-rock heterogeneity measured by borehole logging to the occurrence of seismic events caused by hydraulic fracturing of the corresponding rock sections. Our observations made from two hydraulic fracturing case studies suggest that elastic-rock heterogeneity controls the occurrence of fluid-injection-induced seismicity. The seismic events occurred preferentially in rock sections characterized by low Poisson’s ratio and high Young’s modulus. Based on analytic solutions of stress fluctuations in elastically heterogeneous media in equilibrium to a homogeneous far-field stress, we quantified the relation between elastic-rock heterogeneity and stress changes leading to fracture opening and reactivation in two end member models of 1D and 3D heterogeneity. We found that significant fluctuations of rock stress originated from elastic rock heterogeneity. Moreover, we found that stress changes leading to fracture opening and reactivation in rocks undergo scale invariance spatial fluctuations. This gave a possible physical explanation for the observed scale invariance of seismogenic processes. We analyzed the physical meaning of a heterogeneity index of rocks, which indicated rock sections of high Young’s modulus and low Poisson’s ratio. We found that this index can be used to identify rock sections of high differential stress. In addition, our results suggest that it is related to the occurrence probability of brittle rock failure during hydraulic fracturing. Nevertheless, we found that fluctuations of Poisson’s ratio showed an even stronger correlation to critical stress changes which resulted in opening and reactivation of fractures in rocks. We obtained an analytic solution of stress fluctuations in vertical transverse isotropic layers such as shale deposits and applied it to a hydraulic fracturing case study of a shale gas reservoir. Also in this case, we observed strong relations between seismicity and fluctuations of stress in space.

**INTRODUCTION**

The injection of pressurized fluids through boreholes to stimulate hydrocarbon and geothermal reservoirs, is associated with the occurrence of seismic events. This provides a unique opportunity to study different aspects of rock stress fluctuations during the process of earthquake nucleation and rupturing under well-known and even adjustable conditions. The existence of boreholes at fluid injection sites opens up the possibility to measure in situ physical rock properties and their fluctuations over a broad range of scales. Moreover, borehole breakouts, drilling-induced tensile fractures, and hydraulic fracturing tests provide reliable in situ stress estimates (e.g., Zoback, 2010). In addition, the driving forces leading to fluid-injection-induced seismicity, that is, the mass and pressure of the injected fluid, are known and controllable parameters. This allows us to quantitatively analyze the underlying processes using physics-based models. For instance, a reconstruction of pore pressure changes triggering seismicity during fluid injection in sedimentary and crystalline rocks by Rothert and Shapiro (2007) suggests that reduction of effective normal stress in the range of \(10^3\)–\(10^6\) Pa triggered the analyzed seismicity. This supports the concept of a critically stressed earth’s crust and suggests that significant heterogeneity of rock stress exists. In general, the hydraulic energy, added to the system by the injection of pressurized fluids, results in an increase of pore pressure in the connected, fluid-saturated pore, and fracture space of rocks (e.g., Pearson, 1981; Shapiro et al., 1997; Zoback and Harjes, 1997). In turn, this causes a decrease of the effective normal stress and a destabilization of the rock in consequence. If the frictional strength of preexisting fractures or the tensile strength of the intact rock is exceeded, brittle rock failure and associated seismic events occur. Seismicity induced by fluid injections at any location will fall...
somewhere between these two end member causes, namely, shear reactivation and tensile opening (Shapiro and Dinske, 2009). Shear reactivation of preexisting fractures usually occurs due to fluid injections at geothermal locations, aiming to create enhanced geothermal systems (EGS) in the crystalline basement. During this type of hydraulic stimulation, the injection pressure stays below the minimum effective principal stress $\sigma_2$ at the depth of injection. Because no new macroscopic fractures can be opened under compressive stress, the fluid-rock interaction is approximately linear (e.g., Langenbruch and Shapiro, 2010). During the hydraulic fracturing of hydrocarbon reservoirs in low permeable sedimentary rocks, the injection pressure usually exceeds the large-scale minimum effective principal stress magnitude, and macroscopic tensile fractures are opened. This results in a significant increase of the rocks’ permeability and for this reason in a highly nonlinear fluid-rock interaction (e.g., Hummel and Shapiro, 2012). However because stress changes leading to shear reactivation can be much lower than stress changes leading to the opening of new fractures, the seismic event cloud extends into the rock, beyond the opened hydraulic fracture lengths and widths (Evans et al., 1999). Studies of fluid-injection-induced seismicity from various case studies demonstrate that almost all events show double-couple source mechanisms, corresponding to shear motion on planar faults. However, few tensile events have been identified by the inversion of moment tensors (e.g., Zhao et al., 2014), which mathematically represent the movement on a fault during an earthquake. It is natural to assume that the occurrence probability of seismic events at a given position in the unperturbed reservoir rock is controlled by the in situ state of stress, which defines the magnitude of critical stress changes necessary to open new and reactivate preexisting fractures at this location. If rock stress heterogeneity in the unperturbed reservoir rock is important for the seismogenesis of fluid injection-induced seismicity, it will determine the spatial location and number of seismic events caused by injection of pressurized fluid. Consequently, rock stress heterogeneity could explain the observation that fluid injection-induced seismicity often is restricted to certain depth sections and shows strong variability in the spatial occurrence. However, the underlying cause and the strength of rock stress fluctuations, which eventually control the failure process, are still uncertain. The main purpose of our study is to quantify critical stress changes in the unperturbed reservoir rock. We perform the quantification based on the only directly accessible evidence of rock heterogeneity, which is given by fluctuations of physical-rock properties determined by borehole logging. We analyze two hydraulic fracturing case studies. Based on analytic solutions of stress fluctuations in layered linear elastic media (see Bourne, 2003), we quantify rock stress fluctuations originating from the elastic-rock heterogeneity measured along the treatment wells. The end member model of 1D elastic heterogeneity in a horizontally layered medium should be a good approximation of sedimentary rocks, which are characterized by a much smaller degree of elastic heterogeneity in the horizontal than in the vertical direction. We compute $\sigma_2$ and the Coulomb failure stress (CFS) as indicators of fracture opening and reactivation probability and analyze if correlations to fluctuations of elastic properties measured along the treatment well exist. In addition, we consider a second end member model of elastic heterogeneity, in which we assume that the degree of elastic heterogeneity in the vertical and horizontal directions is equivalent. This model is, for instance, representative for crystalline rocks, which are present at EGS and deep scientific boreholes. Because all rocks show a certain degree of horizontal heterogeneity, stress fluctuations will fall somewhere between the two analyzed end member models of 1D and 3D heterogeneity. Recently, a brittleness index (BI) of rocks has been proposed (Grieser and Bray, 2007; Rickman et al., 2008). Although the physical meaning of the BI is unclear, it is frequently applied in the hydrocarbon industry. We analyze if we can observe and physically justify any relation between the BI and brittle rock failure caused by hydraulic fracturing of sedimentary reservoir rocks. Even though the BI is based on isotropic elastic properties, it has been originally introduced for application to unconventional shale reservoirs, which show a high degree of elastic anisotropy. We extend the analytic solutions of stress fluctuations in heterogeneous linear elastic media (Bourne, 2003) to the case of vertical transverse isotropic (VTI) layers. We apply the solutions to hydraulic fracturing of a shale gas reservoir located in the Horn River Basin, Northeastern British Columbia, Canada (e.g., Dunphy and Campagna, 2011). It has been observed that elastic rock heterogeneity determined from analysis of borehole logs is of a scale-invariant nature (e.g., Evans et al., 1999; Langenbruch and Shapiro, 2014). Moreover, analysis of borehole breakouts suggests that stress in rocks shows scale-invariant fluctuations (Shamir and Zoback, 1992; Day-Lewis et al., 2010). However, the physical origin of scale-invariant stress fluctuations and the relation to elastic-rock heterogeneity are unclear. If stress changes leading to fracture opening and reactivation in rocks undergo verifiable scale-invariant spatial fluctuations, it would provide a physical explanation for scale invariance of seismogenic processes. Moreover, our results would be transferable to other scales of brittle rock failure. Our results will provide a basis to integrate physics-based geomechanical models of rock stress heterogeneity into the research field of fluid-injection-induced seismicity.

**ROCK BRITTLENESS CONCEPTS**

There are various definitions of material brittleness in literature (e.g., Perez Altamar, 2013). Most commonly, a material is defined as brittle if it breaks after experiencing only small deformation. It means that, if stress is applied to a brittle material, it breaks without absorbing much strain energy prior to failure. However, there are brittleness concepts, which differ from this commonly accepted physical definition. Brittleness of rocks, for instance, has been defined by its mineral composition and total organic content (TOC) (Jarvie et al., 2007; Wang and Gale, 2009). Recently, a BI has been defined as a combination of Young’s modulus $E$ and Poisson’s ratio $\nu$ determined from the P- and S-wave traveltime, as well as density logs along boreholes (Grieser and Bray, 2007; Rickman et al., 2008):

$$BI = \left[ \frac{E - E_{\min}}{E_{\max} - E_{\min}} + \frac{\nu - \nu_{\min}}{\nu_{\max} - \nu_{\min}} \right] \times 100 \%,$$

where $E_{\min}$, $\nu_{\min}$, $E_{\max}$, and $\nu_{\max}$ are the minimum and maximum values of $E$ and $\nu$ in the reservoir section of interest. The BI defined according to equation 1 is a relative quantity of a reservoir, and according to Rickman et al. (2008), it describes the rocks’ ability to fail under stress ($\sigma$) and to maintain a fracture ($E$), once the rock fractures. The authors state that zones of high BI in the reservoir, characterized by high $E$ and low $\nu$ values, should be best suitable for...
hydraulic stimulation because they allow the development of a complex fracture network. Table 1 shows the BI of different materials computed according to equation 1. The $E_{\text{min}}$, $\nu_{\text{min}}$, and $\nu_{\text{max}}$ are defined as maximum and minimum values in Table 1, respectively. The resulting brittleness values show that the index given by equation 1 is obviously not an absolute measure for physical brittleness of a material because, for instance, the BI of cork and steel is higher than the BI of glass. The nomenclature of this heterogeneity index of rocks seems to be meaningless. To clarify the physical meaning of the BI, we analyze its relation to brittle rock failure caused by hydraulic fracturing of hydrocarbon reservoirs.

**OBSERVATIONS: CARTHAGE COTTON VALLEY GAS FIELD**

Several hydraulic fracturing experiments were performed in the Cotton Valley gas field in a tight gas sand stone reservoir in 1997. An overview of the operational setup is given in Rutledge et al. (2004). Due to the hydraulic stimulation of the perforated zones in the target formations, numerous seismic events occurred. The locations of the events are shown in map and depth view in Figure 1a. Figure 1b shows the BI computed along treatment wells 21-10 and 21-09. The BI is computed according to equation 1. The values $E$ and $\nu$ are determined from 1 m averaged sonic $P$- and $S$-waves traveltime, as well as density logs along the vertical treatment boreholes. We concentrate our analysis on the depth range (2600–2710 m) of stage-A (21-10) and stage-C (21-09) treatments. Seismic events induced by the hydraulic stimulation were registered and located with high precision. Absolute depth errors, attributed to velocity-model uncertainty, are as great as 4 m, based on misalignment of the source locations with the targeted (perforated) sand intervals (Rutledge and Phillips, 2003). The gray shaded areas in Figure 1b mark the perforated intervals. Brittle rock failure and the associated seismicity are triggered by pore pressure and stress change caused by the injection of pressurized fluid. Because these stress perturbations initially occur only along the perforated zones, a comparison between elastic rock heterogeneity and event density with depth is only feasible along the perforations. The event density (gray lines in Figure 1) computed along well 21-10 (21-09) includes only events induced by stage-A (stage-C). As proposed by Rutledge and Phillips (2003) stage-A events of more than 2650 m were shifted down 4 m, and events below 2680 m were shifted up 2 m. The absolute depths of the clusters can be reasonably shifted up or down a few meters due to velocity uncertainties suggested by the magnitude of station corrections (1 ms on average) Rutledge and Phillips (2003). A clear relation between BI and event density is visible along the perforated intervals of the wells. Higher BI along a perforation results in higher density of microseismic events. Observable deviations are in the range of location uncertainties with depth. Our observations suggest that the probability of fracture opening and reactivation is increasing with the BI. In the next section, we introduce a physics-based geomechanical model of rock stress heterogeneity to explain this observation.

**GEOMECHANICS: ROCK STRESS FLUCTUATIONS ORIGINATING FROM ELASTIC ROCK HETEROGENEITY**

In this section, we compute rock stress fluctuations originating from the elastic heterogeneity of the reservoir rock at Cotton Valley and relate it to the occurrence probability of brittle rock failure during hydraulic reservoir stimulation. In general, brittle rock failure and the associated seismicity during hydraulic reservoir stimulation

<table>
<thead>
<tr>
<th>Cork</th>
<th>Steel</th>
<th>Glass</th>
<th>Cotton Valley rock formation</th>
</tr>
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<tbody>
<tr>
<td>Young’s modulus (GPa)</td>
<td>18.6</td>
<td>210</td>
<td>50–90</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.0</td>
<td>0.3</td>
<td>0.2–0.3</td>
</tr>
<tr>
<td>Brittleness index (%)</td>
<td>50</td>
<td>50</td>
<td>8–35</td>
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Figure 1. (a) Source locations of microseismic events for treatments A and C. The tops and bottoms of the perforated intervals are marked with dashed lines in depth views (Figure taken from Rutledge et al., 2004). (b) Comparison of the density of induced microseismic events with depth and the BI along wells 21-10 and 21-09 at the Cotton Valley gas field. The BI is plotted in black and the event density in gray. A comparison between elastic heterogeneity and event density with depth is only feasible along the perforations. Higher BI along a perforation (gray shaded areas) results in a higher density of microseismic events.
occur due to tensile opening of new and shear reactivation of preexisting fractures. Opening of new fractures can only occur if the minimum effective principal stress $\sigma_1$ is becoming tensile due to pore pressure and stress changes caused by the injection of pressurized fluids. The reactivation probability of preexisting fractures is given by the CFS (e.g., Jaeger et al., 2007)

$$\text{CFS} = S_0 - \tau + \mu (\sigma_n - p_p).$$

(2)

The CFS is associated with the Mohr Coulomb failure criterion and describes the amount of stress change necessary to overcome the frictional strength of a preexisting fracture of a given orientation in a given stress field. The $\tau$ and $\sigma_n$ are shear and normal stress acting on the fault plane, and $S_0$ is the cohesion, that is, the shear stress necessary to initiate failure in absence of normal stress. The $p_p$ represents the pore pressure, and $\mu$ is the coefficient of internal friction. A fault is in a stable state if CFS $> 0$. A fault is unstable if CFS $< 0$ and failure occurs if CFS $= 0$. We compute the CFS according to fractures optimally oriented for failure. The resulting values of CFS correspond to the minimum change of stress from which fracture reactivation is possible. We assume a friction coefficient of $\mu = 1.0$ and cohesion $S_0 = 5$ MPa (see Fischer and Guest, 2011) for our computations. To solely analyze the result of measurement-based elastic rock heterogeneity on fracture strength, we keep friction and cohesion constant. Moreover, this is done because no direct measurements of heterogeneity of these parameters are available. Figure 2a–2c shows Poisson’s ratio $\nu$, Young’s modulus $E$, and BI computed from 1 m averaged P- and S-waves traveltime and density logs along treatment borehole 21-10 at Cotton Valley. We averaged the raw data in 1-m intervals to account for the inherent averaging of the logging process over the active length of the tool (approximately 1 m). Because fluctuations of smaller scale than the active length of the tool cannot be resolved by the logging process, we eliminate these apparent small-scale fluctuations, which are physically not justified. The distribution of $\nu$ and $E$ with depth is highly heterogeneous. The heterogeneity should be caused by changes of mineral composition and structure (e.g., Watt et al., 1976), slip on crossing faults (e.g., Hickman and Zoback, 2004), damage zones (e.g., Faulkner et al., 2006), and other geologic structures (e.g., Martin and Chandler, 1993). Most of these processes are not restricted to certain lithological layers. Faulkner et al. (2006) analyze the relation between microfracture density and elastic properties in fault zones and laboratory experiments on rock samples. Their results show that fluctuations of Young’s modulus and Poisson’s ratio in rocks are governed by changes of the damage level. In particular, they find that increasing microfracture damage strongly affects the elastic properties of rocks, where Young’s modulus is decreasing and Poisson’s ratio is increasing with the level of damage. It suggests that elastic rock heterogeneity measured in single lithological units for which changes of the mineral composition can be excluded could originate from changes of the damage level at

![Figure 2](image-url)

Figure 2. Poisson’s ratio $\nu$, Young’s modulus $E$, and BI computed from 1 m averaged P- and S-wave traveltime, and density logs along well 21-10. The values $\nu$ and $E$ show an inverse correlation characterized by a correlation coefficient of $P_{\nu,E} = -0.22$. The gray-shaded areas correspond to perforated zones. (d and e) PDF of $E$ and $\nu$ obtained from panels (a and b). The $\langle E \rangle$, $\sigma_E$, and $\nu$, $\sigma_\nu$ correspond to the mean value and standard deviation of the best-fitting Gaussian distributions. (f and g) PSD of $E$ and $\nu$ log data. The PSD characterizes the spatial correlation of $\nu$ and $E$. 
different scales. As is observed for most rocks, we find that fluctuations of $\nu$ and $E$ at Cotton Valley are inversely interrelated (see Figure 2a and 2b). We quantify this relation by calculating Pearson's correlation coefficient $Pc$ given by the covariance of $\nu$ and $E$ divided by the product of their standard deviations. The moduli show an inverse relation quantified by $\frac{Pc}{E}$ divided by the product of their standard deviations. The identified inverse development of Young’s modulus and Poisson’s ratio with increasing damage level (see Faulkner et al., 2006) may cause this inverse interrelation. In any case, measurements of elastic properties along boreholes incorporate all processes, which finally resulted in elastic-rock heterogeneity existing in the present-day stress field. To compute the stress fluctuations originating from elastic rock heterogeneity, we consider a perfectly horizontally layered linear elastic medium in equilibrium to a homogeneous far-field stress (see Figure 3). The individual layers are characterized by different values of $E$ and $\nu$. Analytic solutions of stress fluctuations in layered media consisting of isotropic linear elastic layers are given in Bourne (2003) (see Appendix A). We combine this model to in situ rock heterogeneity determined by borehole measurements for the first time. Moreover, we test its applicability and verification by comparing the resulting stress fluctuations with the occurrence of fluid-injection-induced seismicity. These seismic events should be reliable indicators of in situ stress. We do not interpret the borehole measurement data or analyze core samples to introduce layers of different thickness into the model. Instead, we consider each measurement of Young’s modulus and Poisson’s ratio along well 21-10 (see Figure 2a and 2b) as isotropic elastic properties of one elementary layer of 1 m thickness, to account for as many scales as possible. In agreement with Bourne (2003), we assume that the layers characterized by different elastic properties are perfectly coupled. This enforces all layers to undergo the same amount of strain in the horizontal direction and results in fluctuations of the horizontal principal stresses. The degree of elastic heterogeneity in the vertical direction will control the magnitudes of fluctuations of the horizontal stresses. Because no elastic heterogeneity in the horizontal direction is present in a perfectly horizontally layered medium, the vertical stress will be constant throughout the model medium to achieve equilibrium to the far-field stress (see Appendix A and Bourne, 2003). Moreover, we assume that the layered medium is stress free before application of the external stress field. It means that residual stress, which should exist in rocks because lithification took place under certain nonzero stress conditions, is neglected. The stress fluctuations computed under this assumption will be the upper bound of possible stress fluctuations resulting from elastic rock heterogeneity. The far-field stress that we apply to the layered medium is determined from stress and pore pressure profiles at Cotton Valley (see Fischer and Guest, 2011). The stress regime corresponds to normal faulting. Because we do not have reliable information on the magnitude of the maximum horizontal principal stress $\sigma_2$, we assume that its magnitude is the mean value between $\sigma_1$ and $\sigma_3$. The principal stress components of the externally applied stress field are defined according to

$$\sigma_{1e} = 37.1 \text{ MPa} = S_v, \quad \sigma_{2e} = 21.5 \text{ MPa} = S_H, \quad \sigma_{3e} = 6.5 \text{ MPa} = S_h. \quad (3)$$

These stress magnitudes correspond to effective principal stress components in the depth of the considered rock section (2650 m). Compressive stress is always defined positive in the following. We assume that the external stresses act normal to the boundary surfaces of the model. This means that no shear stresses exist at the boundaries. As a result, the direction of principal stresses will be constant in the layered medium. Figure 4 presents the stress fluctuations resulting in the layered medium corresponding to elastic rock heterogeneity characterized along well 21-10. The elastic heterogeneity at Cotton Valley results in significant fluctuations of the horizontal principal stress magnitudes in the different elementary layers. The Mohr circles between the minimum and maximum principal stress are color coded according to the BI (equation 1) of the corresponding rock section. A clear classification is visible. Rock sections characterized by high Young’s modulus and low Poisson’s ratio show low magnitudes of minimum horizontal principal stress and correspondingly contain a high level of differential stress ($\sigma_1 - \sigma_3$). This results in the highest ratio of shear to normal stress on fracture planes located in these layers. Thus, the stress changes necessary to open new (\(\sigma_1\)) and to reactivate preexisting fractures (CFS) are lowest in these layers. According to our model, brittle rock failure and the associated seismicity during hydraulic reservoir stimulation is most probable in rock sections characterized by a high Young’s modulus and a low Poisson’s ratio. These rock sections are characterized by a high BI value. This is confirmed in the histogram plot (Figure 4b and 4c), which represent the number of layers in a given range of minimum principal stress $\sigma_3$ and CFS. The individual bars are color coded according to the mean BI of the layers in the corresponding stress level. As already discussed, $\sigma_3$ and CFS are indicator of fracture opening and reactivation probability and are representing the occurrence probability of seismic events during hydraulic reservoir stimulation. The lower $\sigma_3$ and CFS are in a layer, the lower is the stress perturbation necessary to trigger seismic events in the corresponding layer. The relation

Figure 3. Schematic illustration of a perfectly horizontally layered medium consisting of elastically homogeneous layers. We analyze stress fluctuations that result in the equilibrium state to an applied homogeneous far-field stress at the boundaries of the medium. The magnitudes of the far-field stress are given in the figure. We assume that the layers characterized by different elastic properties are perfectly coupled. This enforces all layers to undergo the same amount of strain in the horizontal direction and results in fluctuations of the horizontal principal stresses in the individual layers.
between elastic rock heterogeneity and the occurrence probability of seismicity resulting from our theoretical considerations are in agreement with the observation of a positive relation between the BI and density of seismic events induced by hydraulic stimulation at the Cotton Valley gas field (see Figure 1). However, other mechanical properties might explain the observations of the correlation between event occurrence and differential stress as well. For example, corresponding with this correlation is a tendency for the stiffer layers to contain and sustain natural fracture development. Possibly there are more potential microseismic sources and associated permeability paths along which pore pressure can extend into the rock. To clarify this point, the controlling mechanisms of the site-specific elastic heterogeneity have to be analyzed. As mentioned above, different mechanisms like mineral composition, structure, damage level, and other geologic structures contribute to the elastic heterogeneity of rocks. Nevertheless, the obtained range of fracture opening and reactivation stress is in agreement with critical stress changes reconstructed from microseismicity in sedimentary and crystalline rocks (see Rothert and Shapiro, 2007). Our results suggest that elastic rock heterogeneity controls the occurrence probability of brittle rock failure and associated seismic events during hydraulic reservoir stimulation. The high stress contrast in the vertical direction restricts the occurrence of seismicity to certain depth ranges and explains the clear separation of event clouds induced along the different perforations. To separately analyze the influence of Young’s modulus and Poisson’s ratio fluctuations on rock stress heterogeneity, we compare stress and elastic property fluctuations. The comparison is presented in Figure 5. In agreement with the theory discussed in Appendix A, we find that rock stress fluctuations are controlled by deviations of in situ elastic moduli from their mean values. We quantify the correlation strength between stress and elastic-moduli heterogeneity, according to the correlation coefficient $P_c$, shown at the bottom of the figure. During interpretation of the comparison, we have to keep in mind that $E$ and $\nu$ are inversely interrelated in the Cotton Valley reservoir rock (see Figure 2a and 2b). It means that, for instance, rock sections characterized by a high Young’s modulus will most probably be characterized by a low value of $\nu$. Although the influence of $\nu$ and $E$ on stress heterogeneity can be quantified using equation A-6, considering the measured fluctuations of elastic properties is important to understand stress fluctuations in real rocks. However, this means that the obtained correlations between stress and elastic property fluctuations are only valid for the analyzed case study, which is characterized by a site-specific elastic rock heterogeneity and far-field stress magnitudes. The mean values, standard deviations, and correlation strength ($P_c$) of Young’s modulus and Poisson’s ratio at other sites will be different. Moreover, equation A-6 shows that not only the elastic rock heterogeneity but also the differential stress magnitudes of the far-field principal stresses will influence the resulting stress fluctuations. Rock failure indicators, which are based solely on the elastic properties of rocks, cannot have a universal physical meaning because the far-field differential stress...
magnitudes influences the strength of heterogeneous materials as well. Comparison between rock stress and elastic heterogeneity (Figure 5) reveals that fluctuations of $\sigma_3$ and CFS in the Cotton Valley reservoir rock are strongly correlated to fluctuations of Poisson’s ratio ($P_c = 0.96$). Moreover, the BI shows a strong inverse correlation to CFS and $\sigma_3$ ($P_c = -0.92$). This provides a physical explanation for the observed relation between event density with depth and the BI (Figure 1). Nevertheless, our results suggest that considering only fluctuations of Poisson’s ratio is the best indicator for the probability of fracture opening and reactivation and associated seismic events at the Cotton Valley reservoir. Our model of stress fluctuations in a perfectly horizontally layered medium in equilibrium to a homogeneous far-field stress seems to be a good representation of rock stress heterogeneity and resulting probability of fracture opening and reactivation. The obtained critical stress changes leading to fracture opening and reactivation along well 21-10 at Cotton Valley show low magnitudes. We note that the computed critical stress changes (CFS and $\sigma_3$) represent the lowest threshold. This is the case because we assumed that the layered medium is stress free before application of the external stress field. Moreover, the CFS is computed for optimally oriented preexisting fractures. Thus, the obtained values of CFS characterize the minimum stress changes from which reactivation of preexisting fractures is possible. Higher stress changes are required to reactivate more unfavorably oriented preexisting fractures. In the same sense, the value of $\sigma_3$ represents the lowest threshold of stress changes causing the opening of new fractures because we neglect the tensile strength of the rocks. If the tensile strength of the rocks is known, the fracture opening stress can be computed according to the Griffith criterion (e.g., Jaeger et al., 2007). Nevertheless, the distribution of CFS and $\sigma_3$ computed from our model is representative for the spatial heterogeneity of the fracture reactivation and opening probability during hydraulic reservoir stimulation.

**SCALE INVARIANCE OF CRITICAL STRESS CHANGES**

It has been observed that heterogeneity in rocks is of a scale-invariant nature (e.g., Leary, 1997; Day-Lewis et al., 2010; Langenbruch and Shapiro, 2014). It means that no characteristic scales exist above or below which the rock heterogeneity can be neglected. Figure 2f and 2g presents the power spectral density (PSD) of $E$ and $\nu$ fluctuations measured along borehole 21-10 at Cotton Valley (see Figure 2a and 2b). The power law dependence of the PSD functions on the wavenumber implies the scale-invariant fractal nature of the elastic rock heterogeneity at Cotton Valley. The power law exponent of $\beta \approx 1$ is in agreement with the observation of universality of the heterogeneous complexity of rocks (e.g., Leary, 1997; Langenbruch and Shapiro, 2014). Our layered model thus shows scale-invariant fluctuations of Young’s modulus and Poisson’s ratio in the vertical direction. Figure 4d and 4e presents the PSD of $\sigma_3$ and CFS fluctuations with depth, which we computed in the layered medium. The PSD of the stress fluctuations is characterized by a power law dependence on the wavenumber. Our results suggest that scale-invariant fluctuations of elastic properties result in scale-invariant fluctuations of the fracture opening and reactivation probability. The power law exponent ($\beta = -0.99$), which is related to the ratio between small- to large-scale fluctuations of stress, is in agreement with the power law exponent of the PSD of elastic-moduli fluctuations (see Figure 2f and 2g). The existence of scale-invariant fluctuations of critical stress changes in rocks provides a possible physical explanation for the observed scale invariance of seismogenic processes. Scale invariance of seismogenic processes is, for instance, expressed in the Gutenberg-Richter relation of earthquake magnitude scaling. Langenbruch and Shapiro (2014) show that the emergence of the Gutenberg-Richter relation can be explained by the existence of scale-invariant rock stress fluctuations. Scale invariance suggests the existence of elastic rock heterogeneity and corresponding rock stress fluctuations at all scales. However, our model only accounts for heterogeneity down to the scale of the active length of the logging tool (approximately 1 m) and up to the length of the considered rock section (300 m). Nevertheless, if scale invariance is fulfilled, the resulting stress fluctuations are valid on other scales. If, for instance, 300 measurements of stress or elastic properties are made along well 21-10 at Cotton Valley on a constant but arbitrary scale, the result will be in agreement with the distributions shown in Figures 2 and 4. The acquired data will be characterized by the same mean values, standard deviations, and power spectra of fluctuations. The maximum magnitude of measured fluctuation of stress and elastic properties, however, will increase with the number of measurements. This means that it is important to consider the scale on which elastic heterogeneity is obtained and the scale of the brittle failure process for which critical stress changes should be quanti-

![Figure 5. Comparison of elastic rock heterogeneity measured along well 21-10 at Cotton Valley and stress fluctuations resulting in a perfectly horizontally layered medium in equilibrium to an applied far-field stress. The strength of the correlation between stress and elastic moduli fluctuations is quantified by the correlation coefficient $P_c$ shown at the bottom of the figure. Stress fluctuations are controlled by deviations of in situ moduli from their mean values. Fluctuations of Poisson’s ratio show the strongest correlation to fluctuations of $\sigma_3$ and CFS. This suggests that fluctuations of Poisson’s ratio are the best indicator for the probability of fracture opening and reactivation during hydraulic stimulation at Cotton Valley.](image-url)
fied. The range of moment magnitudes \((M_w = -2.5 \text{ to } -1.5)\) of seismic events at Cotton Valley suggests that detectable events are characterized by source radii ranging from 0.35 to 1.1 m. This estimation is based on calculation of source radii according to Brune (1970) and Kanamori (1977) considering a constant stress drop of 2.3 MPa (e.g., Goertz-Allmann et al., 2011). This scale is comparable with the scale of the active length of the logging tool. However, it is unclear on which scale the rupture process is initiated. Moreover, scale invariance suggests that failure processes on smaller scales exist, but they are not detectable by the seismic monitoring network at Cotton Valley. If scale invariance is fulfilled, fluctuations of CFS and \(\sigma_2\) on an arbitrary scale can be computed by drawing the corresponding number of independent random samples from the probability density function (PDF) of the CFS and \(\sigma_2\) distribution shown in Figure 4b and 4c. This will be helpful for the further development of physics-based statistical models of fluid-injection-induced earthquakes (e.g., Rothert and Shapiro, 2003; Langenbruch and Shapiro, 2010; Goertz-Allmann and Wiemer, 2013).

We use the best-fitting Gaussian distribution to compute the expected magnitudes of CFS fluctuations on different scales ranging from \(10^{-3}\) to \(10^0\) m (see Figure 6). We consider the 300-m-long borehole section shown in Figure 2. For instance, CFS fluctuations on the centimeter scale are determined by drawing 30,000 independent random samples from the Gaussian distribution shown in Figure 4c. The scale-invariant nature of elastic rock heterogeneity makes a prediction of absolute stress magnitudes at a given depth implausible. The distributions of critical stress changes obtained in our study should be considered as probability density distributions. Absolute values of critical stress changes obtained by our and other studies should be viewed with caution. Measurements of stress in the considered depth are the only reliable method to obtain absolute values of stress magnitudes. However, also in the case of stress measurements, the scale of the rock-failure process, which is used to draw conclusions about stress magnitudes, will be important. Mayerhofer et al. (2010) present some direct stress measurements along well 21-09. The stress measurements are based on hydraulic fracturing stress tests. In the reservoir depth section considered in our study, they find stress fluctuations of approximately 1.33 MPa. The authors state that this stress contrast is too small to explain the confinement of the hydraulic fracture in the targeted sand formation. However, these results are based on only two direct stress measurements. Our results suggest that larger stress contrasts exist on smaller scales. This may explain the confinement of the hydraulic fractures.

**STRESS FLUCTUATIONS IN 3D HETEROGENEOUS MEDIA**

We analyzed stress fluctuations in perfectly horizontally layered media. However, it is clear that all rocks show elastic heterogeneity in the horizontal direction. Therefore, we consider a second end member model of elastic heterogeneity. Here, we assume that the degree of elastic heterogeneity in the vertical and horizontal directions is equivalent. Our results will help to understand the influence of horizontal heterogeneity. Measurements of elastic properties are available along two vertical treatment wells (see Figure 1) at Cotton Valley. We compute the correlation coefficients of fluctuations of \(E\) and \(\nu\) measured along the two wells. The highest correlation coefficients \((P_{C,\nu,\nu} = 0.34\) and \(P_{C,E,E} = 0.62\) result for a layer tilt of 1.53° from horizontal. The tilt of the layers is sufficiently small to be neglected. The high values of the correlation coefficients show that stress fluctuations in a perfectly horizontally layered medium should be a good approximation for rock stress heterogeneity at Cotton Valley. Because no stress fluctuations occur within a single layer, but a strong contrast of critical stress changes exists between different layers, seismic event clouds in sedimentary rocks are often restricted to certain depth ranges. However, our results show that even the sedimentary rock at Cotton Valley shows elastic heterogeneity in the horizontal direction. Figure 7 presents 3D heterogeneous media, which we simulated according to the elastic heterogeneity along well 21-10 (see Figure 2). The media consist

![Figure 6. CFS fluctuations along the 300-m-long borehole section at Cotton Valley on different scales. The histogram of stress fluctuations along the considered rock section of 300 m on the different scales is determined by drawing the corresponding number of independent random samples from the PDF of CFS (see Figure 4c). Because larger stress fluctuations occur on smaller scales, more small magnitude events are triggered.](image)

![Figure 7. Simulated heterogeneous distributions of \(\nu\) and \(E\). The distributions are statistically equivalent to the log data along well 21-10 (see Figure 2), that is, they show the same mean values, standard deviations, and correlation coefficient between \(\nu\) and \(E\). Moreover, the values along any 1D profile in the simulated media possess a power spectrum of fluctuations, given by \(k^{-1}\). Due to the applied Kriging procedure, the distribution of \(\nu\) and \(E\) along the center \((z\text{-axis})\) of our model medium corresponds to measured values along well 21-10.](image)
of equally sized cubes characterized by different values of $E$ and $\nu$ (for details on the simulation procedure, see Langenbruch and Shapiro, 2014). Additionally, we applied Kriging to the simulated distributions of $\nu$ and $E$ to condition the media to measured values along well 21-10. We applied a variation of the simple Kriging method according to Huang et al. (2011). The resulting distribution of $\nu$ and $E$ is statistically equivalent to the elastic heterogeneity along well 21-10. It means that $\nu$ and $E$ show the same mean value, standard deviation, and correlation coefficient as the measured data (see Figure 2). Moreover, the fluctuations along any 1D profile through the simulated media possess a power spectrum of fluctuations given by $k^{-1}$. Due to the applied Kriging procedure, the distribution of $\nu$ and $E$ along the center ($z$-axis) of our model medium corresponds to measured values along well 21-10. In Appendix B, we present new analytic solutions of the stress state in the individual cells of the 3D heterogeneous medium (equation B-2). In agreement with the analysis of a layered medium, we assume that the elements of the different elastic properties are perfectly bounded. This means that the strain inside the medium will be homogeneous, if a far-field stress is applied at its boundaries. We consider the normal faulting far-field stress regime at Cotton Valley. Figure 8a and 8b shows the spatial distribution of $\sigma_z$ and $CFS$ in the complete 3D heterogeneous medium. In Figure 8c–8e, the stress fluctuations along the $z$-axis of the medium are presented. Because heterogeneity in all three directions in space is present, all three principal stress magnitudes undergo fluctuations. The comparison of stress fluctuations and elastic rock heterogeneity (see Figure 9) along well 21-10 reveals that the maximum principal stress magnitude ($\sigma_1$) is governed by fluctuations of Young’s modulus. In general, we find that the relations between the fracture opening and reactivation probability and in situ elastic properties are in agreement with the results obtained in a layered medium. Nevertheless, the correlation strength between the BI, $\sigma_z$, and CFS is weaker than in the layered medium. This is so because the maximum principal stress undergoes significant fluctuations due to the existence of horizontal elastic heterogeneity. The color coding in Figure 8 and the correlation coefficient of $P_C = 0.93$ reveals that the BI is a very strong indicator of differential stress. In agreement with the results obtained in the layered medium, fluctuations of $\nu$ seem to be the best probability indicator of brittle rock failure and the associated seismicity during hydraulic reservoir stimulation. The results obtained for the isotropic 3D heterogeneous medium will also be valid in a strike-slip and reverse-faulting stress regime because interchanging the stress directions has no effect on the correlations. The maximum principal stress is governed by fluctuations of $E$, and fluctuations of $\sigma_3$ are strongly correlated to $\nu$. The magnitudes of stress fluctuations in the three directions of space in the analyzed 3D heterogeneous medium are equivalent. Because stress at a given depth will strongly fluctuate in the horizontal direction, no restriction of seismicity to a certain depth section should occur in the 3D heterogeneous case. Crystalline rocks, which are present at EGS and deep scientific boreholes, such as the Continental Deep Drilling Site (KTB) (Germany) and the San Andreas Fault Observatory at Depth (SAFOD) (USA), are characterized by a comparable degree of heterogeneity in all three directions of space. As a result, stress magnitudes should fluctuate to the same degree in the horizontal and vertical directions. Natural and fluid-injection-induced earthquakes have been observed at the SAFOD and the KTB. Hypocenter locations of these earthquakes and fluid-injection-induced earthquakes at EGS (e.g., Langenbruch and Shapiro, 2010; Langenbruch et al., 2011) show a diffuse spatial distribution in all three directions in space. This is in agreement with the consideration of similar fluctuations of stress in all three directions. A comparison of seismic event occurrence with depth and elastic heterogeneity measured along injection boreholes in crystalline rocks does not seem to be feasible. In situ elastic properties and resulting fluctuations of stress just a few meters away from the borehole are significantly different than the fluctuations measured along the borehole. Nevertheless, our results suggest that rock stress heterogeneity has a significant influence on the seismogenesis of fluid injection-induced seismicity in the 3D heterogeneous case.

**HORN RIVER SHALE GAS RESERVOIR: ROCK STRESS FLUCTUATIONS IN LAYERED VERTICAL TRANSVERSE ISOTROPIC MEDIA**

Because the target sandstone formation at Cotton Valley is characterized by a minor degree of intrinsic anisotropy, the rock formation can be considered as elastically isotropic. However, the BI originally has been introduced for application to unconventional shale reservoirs, which show a high degree of elastic anisotropy. The anisotropy is mainly caused by alignment and lamination of softer and platy clay minerals and kerogen. Usually, this intrinsic anisotropy is described by VTI media. These media are characterized by two different Young’s moduli and shear moduli ($E_h$, $\mu_h$, and $\nu_h$) in the vertical and horizontal directions, respectively. In addition, three different Poisson’s ratios $\nu_{vh}$, $\nu_{hh}$, and $\nu_{hv}$ exist, where the first subscript defines the direction of applied stress and the second subscript gives the direction to which strain is compared. Only five of these seven properties are independent ($\nu_{hh} = \frac{h}{h} \approx 0.5$ and $\nu_{hv} = \frac{h}{h}$) (e.g., Bower, 2014). The theoretical value range of the elastic properties in the VTI case is given by (e.g., Amadei, 1996)

$$
\begin{align*}
E_h, E_v, \mu_h, \mu_v > 0, & \quad -1 < \nu_{hh} < 1, \\
- \frac{E_v (1 - \nu_{hh})}{E_h} < \nu_{hv} & < \frac{E_v (1 - \nu_{hh})}{E_h}. 
\end{align*}
$$

In general, fluctuations of all five independent elastic properties influence the state of stress in elastically heterogeneous VTI media. Thus, it is obvious that the BI, which is defined based on isotropic elastic properties, cannot represent the heterogeneity of shale reservoir rocks. However, unconventional shale reservoir rocks show stratification and strong relations between elastic properties, which have to be taken into account. We analyze a hydraulic fracturing case study at a shale gas reservoir located in the Horn River Basin, Northern British Columbia, Canada. More than 3000 events have been recorded and located during the three-stage hydraulic fracturing treatment through a horizontal borehole. The event occurrence is restricted to a certain depth range, which approximately corresponds to two lithologic layers (see Dunphy and Campagna, 2011). These two organic-rich target shale formations (TOC ≈ 5%) are characterized by a high quartz content (approximately 65%) and a clay content of approximately 27%. The formations are overlain by a 800-m-thick clay-rich (approximately 70%) shale formation. Below the reservoir another clay-rich shale formation (approximately 40%) and a thin carbonate layer are present (see Yu and Shapiro, 2014). The clay-rich shale formations and the thin carbonate
Figure 8. Stress fluctuations in the 3D heterogeneous medium (see Figure 7). (a and b) Spatial distribution of \( \sigma_3 \) and CFS. (c) Mohr circle representation of stress fluctuations along the \( z \)-axis (well 21-10). The stars indicate the shear and normal stress acting on optimally oriented fracture planes, and the color coding corresponds to the BI. The externally applied homogeneous far-field stress is represented by the thick black circles. (d and e) Histogram plot of \( \sigma_3 \) and CFS. The bars are color coded according to the mean BI of the elementary layers in the corresponding stress level.
Figure 9. Relation between elastic properties and computed stress fluctuations along well 21-10 in the 3D heterogeneous medium shown in Figure 7. Correlation between stress and elastic moduli is quantified by the correlation coefficient $P_c$ shown at the bottom of the figure. Stress fluctuations are controlled by deviation of the in situ moduli from their mean values. Fluctuations of Poisson’s ratio show the strongest correlation to the occurrence probability of brittle rock failure and associated seismic events.

Figure 10. Horn River shale gas reservoir: (a-c) $V_p$, $V_s$, and density logs along a vertical well in the study area. (d-f) Isotropic elastic properties and BI computed from panels (a-c). (g) Thomson parameter. (h-j) Anisotropic elastic properties of the VTI medium computed according to panels (a-c). (g) A VTI medium is characterized by five independent elastic properties. The anisotropic elastic properties satisfy the constraints given in equation 4.
layer, seem to be fracture barriers. Density and P- and S-wave traveltime logs are measured along a nearby vertical well in the study area (see Figure 10a and 10c). Based on this information, we compute the isotropic Young’s modulus, Poisson’s ratio, and BI of the shale reservoir with depth (see Figure 10d and 10f). In addition, information about Thomson’s parameters (\(\epsilon, \gamma, \text{ and } \delta\)) is available from analysis of a VSP survey, rock-physical constraints, and inversion of microseismic data (Yu and Shapiro, 2014) (see Figure 10g). The five independent elastic properties (\(E_v, E_h, \nu_{vh}, \nu_{lh}, \text{ and } \mu_v\)) in the VTI case can be computed using \(V_p, V_s, \rho, \epsilon, \gamma, \text{ and } \delta\) (e.g., Grechka, 2009). The \(V_p\) and \(V_s\) correspond to P- and S-wave velocities along the symmetry axis, respectively. Figure 10h–10j presents the resulting values of the anisotropic elastic properties. The anisotropic elastic properties fall in the value range given by equation 4. In Appendix A, we extend the solutions of Bourne (2003) to the case of elastically VTI layers. Based on these new analytic solutions (equation A-6), we are able to compute stress fluctuations in layered VTI media. We consider the distribution of elastic properties corresponding to the Horn River

Figure 11. Stress fluctuations resulting from the VTI elastic-rock heterogeneity of the Horn River shale gas reservoir rock in equilibrium to a homogeneous far-field stress. (a) Mohr circle representation of stress fluctuations. (b and c) Histogram plots of minimum principal stress \(\sigma_3\) and CFS. The bars are color coded according to the mean BI of the elementary layers in the corresponding stress level.

Figure 12. Horn River Basin shale gas reservoir: Stress modeling results in a layered VTI medium created according to the data shown in Figure 10h–10j. From left to right: \(\sigma_3\) magnitude with depth. The black line shows the distribution resulting from the assumption of isotropic elastic properties (Figure 10d and 10e). The gray line shows the results according to a VTI medium (Figure 10h–10j). The second panel shows the density of seismic events with depth. The event density follows the distribution of the minimum principal stress magnitude. An upper and lower fracture barrier corresponding to rock sections in a stable state of stress, restricts the depth range of seismic event occurrence. In the last two panels to the right, we compare the magnitude of \(\sigma_3\) and CFS computed for the VTI medium to the BI computed according to isotropic elastic properties. BI, CFS, and \(\sigma_3\) show a strong inverse correlation.
shale gas reservoir. The far-field stresses, which we apply to the medium are computed as follows: The vertical effective principal stress magnitude is determined from the lithostatic gradient and the assumption of a hydrostatic pore pressure gradient. At the depth of 1775 m (the depth of the perforations), it is given by: \( S_V = 27.61 \text{ MPa} \). We assume that the frictional rock strength constrains the magnitude of the minimum effective stress in the brittle earth’s crust. The minimum principal far-field stress can then be determined according to (see, Zoback, 2010)

\[
\frac{S_V}{S_h} = \left( \mu^2 + 1 \right) + \mu^2,
\]

where \( \mu \) is the coefficient of friction. In agreement with the analysis in the previous section, we assume a friction coefficient of \( \mu = 1 \). This results in a minimum effective horizontal stress magnitude of \( S_h = 4.74 \text{ MPa} \). The intermediate principal effective far-field stress magnitude is assumed to be the mean value between \( S_V \) and \( S_h \) (\( S_m = 16.17 \text{ MPa} \)). Figure 11 shows the Mohr circle representation and the histogram of CFS and \( \sigma_3 \) in the Horn River shale gas reservoir rock. The Mohr circles and histogram plots are color coded according to the value of the BI computed from isotropic elastic properties (Figure 10d). Even though the stress fluctuations are computed according to anisotropic elastic properties, a strong correlation to the isotropic BI value is visible. In agreement with the findings of the isotropic case at Cotton Valley, the probability of fracture opening (\( \sigma_1 \)) and reactivation (CFS) is increasing with the BI. Moreover, the differential stress and the BI show a strong correlation. The bimodal distribution of the histograms of \( \sigma_3 \) and CFS (Figure 11 bottom) results from the strong contrast of elastic properties between the target shale formations and the overlying clay-rich shale formation. The state of stress in the clay rich shale formations is much less critical than the state of stress in the target formations. The depth distribution of the seismic events induced by the hydraulic fracturing is shown in Figure 12. The density of the events with depth follows the distribution of the minimum principal stress magnitude resulting from our computation. Although the shale reservoir is characterized by a high degree of elastic anisotropy, the minimum principal stress magnitude computed according to VTI elastic properties and isotropic elastic properties (Figure 12 left) shows the same trend with depth. This explains the strong correlation between the BI and the computed probability of brittle rock failure in the shale reservoir rock (see Figure 12 right). Moreover, we find that an upper and lower fracture barrier restrict the depth range of seismic event occurrence. The fracture barriers correspond to shale-rich rock sections in a stable state of stress. Our results suggest that even though the analyzed shale reservoir rock is characterized by a high degree of intrinsic anisotropy, isotropic elastic properties will provide an estimate of the occurrence probability of seismic events during hydraulic fracturing of the corresponding rock section.

**CONCLUSIONS**

We analyzed the relations between elastic-rock heterogeneity, rock-stress fluctuations, and occurrence of fluid injection-induced seismicity. Our findings suggest that elastic-rock heterogeneity controls the occurrence probability of seismic events in the rock being hydraulically fractured. The heterogeneity causes significant spatial fluctuations of critical stress changes leading to opening of new and reactivation of preexisting fractures. Moreover, we demonstrated that critical stress changes undergo scale-invariant fluctuations in rocks. The scale-invariant nature of rock-stress fluctuations is caused by scale-invariant fluctuations of elastic-rock properties. This gives a possible physical explanation for the scale invariance of seismic processes. We compare elastic-rock heterogeneity obtained from borehole logs with the spatial occurrence of seismic events caused by hydraulic fracturing of the corresponding rock section. Our comparison revealed that the seismic events occur preferentially in rock sections characterized by low Poisson’s ratio and high Young’s modulus. Based on analytic solutions of stress fluctuations in heterogeneous linear elastic media in equilibrium to a homogeneous far-field stress, we quantify the relation between elastic-rock heterogeneity and critical stress changes. We analyzed two end member models of 1D and 3D elastic heterogeneity. Because all rocks show a certain degree of horizontal heterogeneity, stress fluctuations will always fall somewhere between these two analyzed models. In both cases, stress fluctuations are governed by deviations of in situ elastic properties from their mean values. We analyzed the physical meaning of a heterogeneity index of rocks, which indicates rock sections of high Young’s modulus and low Poisson’s ratio. We showed that this index can be used to identify rock sections of high differential stress. In addition, our results suggest that it is related to the occurrence probability of brittle rock failure during hydraulic fracturing. Nevertheless, we find that fluctuations of Poisson’s ratio show an even stronger correlation to critical stress changes, which result in opening and reactivation of fractures in rocks. We obtained an analytic solution for stress fluctuations in VTI layers, such as shale deposits, and applied it to a hydraulic fracturing case study of an unconventional shale reservoir in the Horn River basin. Also in this case, we observed strong relations between seismicity and fluctuation of stress in space. However, only if approximate magnitudes of the tectonic far-field stresses are known, it is possible to identify proper rock sections for hydraulic stimulation by analysis of elastic-rock heterogeneity measured along boreholes. Our quantitative study demonstrated that rock failure indicators such as the BI, which is based solely on elastic properties of rocks, cannot have a universal physical meaning. This is the case because the far-field differential stress magnitudes influence the fracture strength of heterogeneous materials as well.

**ACKNOWLEDGMENTS**

We thank the sponsors of the Physics and Application of Seismic Emission Consortium project and the Federal Ministry for the Environment, Nature Conservation and Nuclear Safety as a sponsor of project MeProRisk II for supporting the research presented in this paper. We acknowledge J. Rutledge, two anonymous reviewers, and the associate editor for helpful and constructive comments.

**APPENDIX A**

**LAYERED TRANSVERSE ISOTROPIC MEDIUM UNDER A HOMOGENEOUS FAR-FIELD STRESS**

In this appendix, we present analytic solutions of stress fluctuations in heterogeneous linear elastic media consisting of elastically VTI layers in equilibrium to a homogeneous far-field stress. The
solutions are a generalization of the solutions corresponding to elastically isotropic layers given in Bourne (2003). The consideration of VTI layers allows application to intrinsically anisotropic rocks such as shale deposits. This means that the elastic anisotropy of the individual layers considered in our model results from alignment of platy clay minerals and kerogen particles and not from the layering of the complete medium. The effective macroscopic elastic properties of a layered medium, however, will be anisotropic even in the case of isotropic elastic properties of the individual layers. However, intrinsic rock anisotropy usually plays a more important role than the anisotropy caused by the layering. The solutions of Bourne (2003) follow as a special case of our solutions for VTI layers. We assume the z-axis to be the symmetry axis. The elastic moduli of a VTI medium can be expressed by five independent quantities in terms of Young’s modulus ($E_h$), Poisson ratio ($\nu$), and shear modulus ($\mu$)

$$E_h = E_x = E_y, \quad E_v = E_z, \quad \nu_{hh} = \nu_{xy} = \nu_{yx},$$

$$\nu_{eh} = \nu_{cx} = \nu_{cy}, \quad \nu_{hv} = \nu_{xv} = \nu_{yz}, \quad \nu_{ve} = \nu_{hv} = \nu_{ve},$$

$$\mu_h = \mu_{xy} = \mu_{yy} = \frac{E_h}{(1 + \nu_{hh})},$$

$$\mu_v = \mu_{xz} = \mu_{yz} = \mu_{zy},$$

(A-1)

where $h$ and $v$ indicate the horizontal ($x$- and $y$-directions) and vertical ($z$-direction), respectively. The five independent elastic properties can be expressed as $E_h, \nu_{hh}, \nu_{vh}, \mu_v$, and $\mu_h$. These properties can be computed from $V_p, V_S, \rho, \epsilon, \gamma,$ and $\delta$ (e.g., Grechka, 2009). The $V_p$ and $V_S$ correspond to P- and S-wave velocities along the symmetry axis, respectively. The $\rho$ is the density and $\epsilon, \gamma,$ and $\delta$ are the Thomson’s parameter. Hooke’s law for the VTI case reads

$$\begin{bmatrix}
E_{xx} & -\frac{E_{vh}}{E_v} & -\frac{E_{vh}}{E_v} & 0 & 0 & 0 \\
-\frac{E_{vh}}{E_v} & E_{yy} & -\frac{E_{vh}}{E_v} & 0 & 0 & 0 \\
-\frac{E_{vh}}{E_v} & -\frac{E_{vh}}{E_v} & E_{zz} & 0 & 0 & 0 \\
2E_{yz} & 0 & 0 & \frac{1}{\rho_e} & 0 & 0 \\
2E_{zx} & 0 & 0 & 0 & \frac{1}{\rho_v} & 0 \\
2E_{xy} & 0 & 0 & 0 & 0 & \frac{2(1+\nu_{vh})}{E_h}
\end{bmatrix} \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{zx} \\
\sigma_{xy}
\end{bmatrix}.$$ 

In agreement with Bourne (2003), we consider a perfectly horizontally layered medium and perfectly coupled boundaries between the layers. It means that no relative movement parallel to the layer boundaries can occur. Thus, the external application of a homogeneous far-field stress ($\sigma_{ij,e}$) will cause constant layer-parallel strain. Moreover, we assume that the layered medium is stress free before application of the external stress field. This means that residual stress, which should exist in rocks because lithification took place under certain nonzero stress conditions, is neglected. The stress fluctuations computed under this assumption will be the upper bound of possible stress fluctuations resulting from elastic rock heterogeneity. According to Hooke’s law, the layer parallel strain is given by

$$\epsilon_{xx} = \frac{1}{E_{h,i}} \sigma_{xx,i} - \frac{\nu_{hh,i}}{E_{h,i}} \sigma_{yy,i} - \frac{\nu_{vh,i}}{E_{v,i}} \sigma_{zz,i},$$

$$\epsilon_{yy} = \frac{1}{E_{h,i}} \sigma_{yy,i} - \frac{\nu_{hh,i}}{E_{h,i}} \sigma_{xx,i} - \frac{\nu_{vh,i}}{E_{v,i}} \sigma_{zz,i},$$

$$\epsilon_{xy} = \frac{1}{2\mu_{h,i}} \sigma_{xy,i},$$

(A-2)

where the index $i$ indicates the elastic properties or stress magnitudes in the $i$th layer. Now, we must find the horizontal strain that is in equilibrium with the applied far-field stress. The sum of all layer-parallel stresses has to be balanced by the horizontal components of the homogeneous far-field stress at the boundaries. The approach is comparable with effective medium descriptions of finely layered anisotropic media (Backus, 1962; Schoenberg and Muir, 1989), but with the distinction of investigating the internal state of individual layers rather than the overall state of the ensemble (Bourne, 2003). The balance of forces between far-field stress and the coupled layers requires

$$\sigma_{xx,e} \sum_{i=1}^{N} t_i = \sum_{i=1}^{N} \sigma_{xx,i} t_i,$$

$$\sigma_{yy,e} \sum_{i=1}^{N} t_i = \sum_{i=1}^{N} \sigma_{yy,i} t_i,$$

(A-3)

where $t_i$ is the thickness of the $i$th layer. The layer-parallel strain is the same in each layer

$$\sigma_{xx,e} = m_1 \epsilon_{xx} + m_2 \epsilon_{yy} + m_4 \sigma_{zz,e},$$

$$\sigma_{yy,e} = m_1 \epsilon_{yy} + m_2 \epsilon_{xx} + m_4 \sigma_{zz,e},$$

$$\sigma_{xy,e} = m_3 \epsilon_{xy},$$

(A-4)

where $m_1, m_2,$ and $m_4$ are the thickness averaged quantities, where

$$m_{1,i} = \frac{E_{h,i}}{1 - \nu_{hh,i}}, \quad m_{2,i} = \frac{E_{h,i} \nu_{vh,i}}{1 - \nu_{hh,i}}, \quad m_{3,i} = 2\mu_{h,i},$$

$$m_{4,i} = \frac{E_{h,i}}{E_{v,i} \nu_{vh,i} (1 - \nu_{hh,i})}.$$ 

(A-5)

Because we are considering a medium consisting of equally thick layers, the thickness averaged coefficients $m_1, m_2, m_3,$ and $m_4$ are given by the mean values of $m_{1,i}, m_{2,i}, m_{3,i},$ and $m_{4,i},$ respectively. In the case of varying layer thickness, the averaged coefficients are given by the thickness-weighted average quantities evaluated for all $N$ layers (see Bourne, 2003). Eliminating the layer parallel strains considering equation A-4 results in

$$\sigma_{xx,i} = M_1 \sigma_{xx,e} + M_2 \sigma_{yy,e} - M_4 \sigma_{zz,e},$$

$$\sigma_{yy,i} = M_1 \sigma_{yy,e} + M_2 \sigma_{xx,e} - M_4 \sigma_{zz,e},$$

$$\sigma_{xy,i} = M_3 \sigma_{xy,e},$$

(A-6)

where $M_1, M_2,$ and $M_4$ are the thickness weighted average quantities, evaluated for all layers.
where $\sigma_{ij,e}$ are the components of the far-field stress and the coupling coefficients $M_1$, $M_2$, $M_3$, and $M_4$ are given by

$$M_{1,i} = \frac{m_1 m_{1,i} - m_2 m_{2,i}}{m_1 - m_2}, \quad M_{2,i} = \frac{m_1 m_{2,i} - m_2 m_{1,i}}{m_1 - m_2},$$

$$M_{3,i} = \frac{m_{3,i}}{m_3}, \quad M_{4,i} = \frac{m_1 m_{1,i} + m_{2,i}}{m_1 + m_2} - m_{4,i}. \quad (A-7)$$

All stresses involving the index $z$ must be continuous to be in equilibrium to the applied far-field stress:

$$\sigma_{zz,i} = \sigma_{zz,e}, \quad \sigma_{xz,i} = \sigma_{xz,e}, \quad \sigma_{yz,i} = \sigma_{yz,e}. \quad (A-8)$$

The isotropic solutions of Bourne (2003) follow from setting $E_{ij} = E_h = E$, $\mu = \mu_h = \mu$, and $\nu_{ih} = \nu_{hh} = \nu$ in equation A-5 according to

$$m_{1,i} = \frac{E_i}{1 - \nu_i^2}, \quad m_{2,i} = \frac{E_i \nu_i}{1 - \nu_i^2}, \quad m_{3,i} = 2\mu_i, \quad m_{4,i} = \frac{\nu_i}{1 - \nu_i}. \quad (A-9)$$

## APPENDIX B

### 3D HETEROGENEOUS MEDIUM UNDER A HOMOGENEOUS FAR-FIELD STRESS

If a homogeneous far-field stress is applied at the boundaries of a 3D heterogeneous medium consisting of equally sized perfectly bounded cubes, which are characterized by different isotropic elastic properties ($E$ and $\nu$) (see Figure 7) not only the strain in the horizontal, but also the strain in the vertical direction will be homogeneous. This will result in fluctuations of all three principal stresses inside the medium. We use this model to obtain stress fluctuations in 3D elastically heterogeneous media. The strain resulting from application of a homogeneous far-field stress is given by

$$\epsilon_{xx} = \frac{1}{E_{ijk}} \left[ \sigma_{xx,ijk} - \nu_{ijk} (\sigma_{yy,ijk} + \sigma_{zz,ijk}) \right],$$

$$\epsilon_{yy} = \frac{1}{E_{ijk}} \left[ \sigma_{yy,ijk} - \nu_{ijk} (\sigma_{xx,ijk} + \sigma_{zz,ijk}) \right], \quad (B-1)$$

$$\epsilon_{zz} = \frac{1}{E_{ijk}} \left[ \sigma_{zz,ijk} - \nu_{ijk} (\sigma_{xx,ijk} + \sigma_{yy,ijk}) \right],$$

where the subscripts ($ijk$) indicate a cube in the $x$-, $y$-, and $z$-directions. Stress fluctuations resulting in equilibrium to a homogeneous far-field stress are given by

$$\sigma_{xx,ijk} = \frac{E_{ijk} \left[ m_1 (\nu_{ijk} - 1) + m_2 (2 - 3\nu_{ijk}) \right]}{2\nu_{ijk} + \nu_{ijk} - 1},$$

$$m_{1,ijk} = \frac{E_{ijk} \left[ (\nu_{ijk} - 1) + m_2 (1 - 3\nu_{ijk}) \right]}{2\nu_{ijk} + \nu_{ijk} - 1}, \quad m_{2,ijk} = \frac{E_{ijk} \left[ (\nu_{ijk} - 1) - m_2 (1 - 3\nu_{ijk}) \right]}{2\nu_{ijk} + \nu_{ijk} - 1}. \quad (B-3)$$

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