1. INTRODUCTION

In many unconventional hydrocarbon and geothermal reservoirs, fluid flow occurs mainly through networks of fractures and faults that provide conductive connections to wells. It has become common practice in both the petroleum and geothermal industries to attempt to improve the fluid flow properties of a resource through various reservoir stimulation strategies. The goal of any reservoir stimulation treatment is to expose the well to a larger contact area with the reservoir rock, which can be accomplished by creating new fractures through hydraulic fracturing, by improving the permeability of natural fractures through shear stimulation, or both.

In order to optimize production in these types of resources, it is imperative to recognize that fracture networks are dynamic systems, and that their fluid flow properties can evolve as the development of a field progresses. The local state of stress controls the dynamic behavior of the fracture networks. By leveraging a strong understanding of the principals of geomechanics, reservoir engineers are able to develop improved reservoir management strategies during both stimulation and production phases.

In this paper, we present a novel formulation for a numerical reservoir simulator that couples fluid flow, fracture mechanics, and fracture propagation. We extended the model introduced by McClure [1] to include the effects of mass transfer between fractures and the surrounding matrix rock by adopting an embedded fracture modeling (EFM) approach.

This paper is organized as follows. In Section 2, the numerical formulation of the model is described. In Sections 3 and 4, two numerical studies are presented in order to verify the accuracy of the model. The first study involves a “static” fracture problem for which a well known analytical solution is available. The second study demonstrates the adaptive nature of the EFM framework for problems that involve fracture propagation. In Section 5, we draw several conclusions about the utility of the model and discuss potential practical applications for the simulator.

2. NUMERICAL FORMULATION

In this work, we have developed a reservoir simulator that is capable of calculating the coupled interaction between fluid flow in fractures, fluid flow in matrix rock, geomechanics, friction evolution along fracture surfaces, and fracture propagation. The numerical formulations for each of the main modules in the simulator are presented. These three modules are: a) mass transfer, b) geomechanics, and c) permeability evolution, friction, and fracture propagation.

2.1. Mass Transfer: Embedded Fracture Modeling

Modeling fluid flow in highly fractured reservoirs has a rich history in the petroleum and geothermal literature.
The most commonly applied method in practice is probably the double porosity model [2, 3]. This model is applicable if the fracture orientations and lengths are relatively randomly distributed and the fracture system is connected extensively. In the double porosity model, reservoir scale flow occurs in the fractures, and leakoff or charging of fractures occurs locally. The transmissibility between the fracture and matrix domains is calculated based on relatively simple fracture geometry.

In order to honor more realistic representations of fractured reservoir geology, discrete fracture approaches were developed. For example, Karimi-Fard et al. [4] present a discrete fracture model (DFM) in which the geometry of the fractures and faults are captured by discretizing them explicitly in lower-dimensional space, and an unstructured matrix discretization conforms to the fractures. In general, DFM approaches are useful for describing fluid flow behavior in settings where production is dominated by flow through fractures (e.g., formations with low matrix permeability) or if fractures tend to have preferred orientations.

Traditional DFM techniques are subject to several well-documented drawbacks. Creating a matrix discretization that conforms to the fractures often results in a large number of “small” matrix control volumes near fracture intersections. For problems with many fractures, this contributes to significant computational burden. For multiphase flow problems, timestep selection is often controlled by the smallest control volumes, so the fine discretization near fracture intersections is undesirable. Finally, the contrast in permeability between the fracture and matrix domain can differ by several orders of magnitude. This can result in systems of equations that are extremely ill-conditioned, which has a detrimental effect on accuracy and convergence rates of linear solvers [5].

In this work, we have extended a reservoir stimulation simulator that is based on a discrete fracture approach for fluid flow in the fracture domain [1]. Using an explicit representation of fracture networks lends itself well to track the nucleation and propagation of tensile fractures during a stimulation treatment. The original model assumed that matrix permeability was low enough to neglect mass transfer between fractures and the surrounding matrix rock, and therefore only the fractures required discretization.

Traditional DFM techniques are not well suited for problems in which the fracture networks are dynamic and growing over time. In order to model fracture propagation using a DFM, there are two obvious potential options. The matrix domain could constantly be rediscretized during the simulation, or the system could be prediscretized subject to some assumption of where the fracture might potentially propagate. Both options require a significant additional computational burden, and the latter requires implicit assumptions about the physics of fracture propagation.

In the present work, we avoided the use of traditional DFM techniques, and instead adopted an approach called embedded fracture modeling. In the embedded fracture modeling (EFM) approach, the fracture and matrix domains are treated as separate computational domains. The two systems are discretized completely independently (i.e., a conforming mesh is not required; see Fig. 1), and mass conservation is strictly enforced using physics-derived source terms. The EFM approach is conceptually very similar to dual porosity or dual permeability models, but is able to maintain a more realistic representation of geologic features.

![Illustration of the conceptual approach for discretization using the EFM approach. The main advantage is that the matrix grid is not required to conform to the fractures.](image)

The EFM approach was originally introduced by Lee et al. [6, 7], and later expanded upon by Li and Lee [8]. Karvounis [9] developed a heat and mass transfer geothermal simulator based on EFM, and demonstrated that EFM can obtain a suitable degree of accuracy with improved computational performance compared to more traditional simulators. Moinfar et al. [10] compared DFM to EFM for multiphase flow problems, and demonstrated that EFM was able to capture a high degree of accuracy. Moinfar et al. [11] incorporated a simple treatment for calculating fracture permeability evolution due to changes in effective stress in an EFM framework, but stopped short of including a rigorous treatment of geomechanics. Norbeck et al. [12] presented a numerical formulation that integrated the EFM framework into a fully coupled fluid flow, geomechanics, and fracture propagation simulator.

We now present the formulation for the EFM approach. The mass conservation equations are expressed separately for the matrix and fracture domains. For a single-phase fluid, the continuity equations can be written, for flow in the matrix domain, as:

\[
\nabla \cdot (k^m \lambda \nabla p^m) + \rho \dot{q}^m + \bar{\Psi}^m = \frac{\partial}{\partial t} (\rho \phi),
\]

and, for flow in the fracture domain, as:
\[ \nabla \cdot (e k' \lambda \nabla p') + \rho \ddot{q}^{mf} + \ddot{\Psi}^{mf} = \frac{\partial}{\partial t} (\rho E). \quad (2) \]

All variables are described in the Notation section. In addition to the usual terms related to flux, wells, and storage, the terms \( \ddot{\Psi}^{in} \) and \( \ddot{\Psi}^{mf} \) are introduced to allow for mass transfer between the two domains. These mass transfer terms have the following form:

\[ \ddot{\Psi}^{in} = \eta (p' - p^{in})/V, \quad (3) \]

and

\[ \ddot{\Psi}^{mf} = \eta (p^{mf} - p')/A, \quad (4) \]

where the parameter \( \eta \) is called the fracture index and is analogous to the Peaceman well index [13]. The main assumptions involved in the derivation of the fracture index are: a) flow in the vicinity of the fracture is linear, b) the fracture fully penetrates the matrix control volume in the vertical direction, and c) the matrix pressure represents the average pressure over the control volume [8, 9]. Given these assumptions, the fracture index can be shown to be [5]:

\[ \eta = \frac{A'}{\langle d \rangle} k^{mf} \lambda, \quad (5) \]

where \( A' \) is the total surface area of the fracture control volume and \( \langle d \rangle \) represents the average normal distance from the fracture surface in the matrix control volume (see [12] for a derivation of Eq. (5)). It is clear that the fracture index is mainly affected by the matrix permeability and the surface area over which mass transfer occurs, which is a physically intuitive result of the derivation.

With the definitions of the mass transfer terms given in Eqs. (3) and (4), the utility of the EFM approach for fracture propagation problems is revealed. The coupling between the fracture and matrix domains has been fully reduced to a collection of simple source terms, as illustrated in Eqs. (1) and (2). From the perspective of the matrix domain, each fracture element appears essentially like a well. As new fracture elements nucleate and propagate according to mechanics-driven effects, new source terms can be added to the appropriate matrix mass balance residual equations. In this manner, the EFM approach completely avoids the numerical hurdles associated with the constraints imposed by a matrix discretization that must conform to the fractures.

2.2. Geomechanics

The present simulator was built upon the framework introduced by McClure [1] for coupling fluid flow in fractures and fracture mechanics. The reader is referred to McClure [1] and McClure and Horne [14], where thorough descriptions of the assumptions and the numerical formulation for the geomechanics module are explained. Here, we provide a brief overview of the major components of the model.

We are principally interested in modeling systems that contain a large number of fractures and faults, and therefore the displacement fields are expected to be discontinuous. A boundary element method called the displacement discontinuity (DD) method is capable of calculating the complex displacement fields that arise due to the mechanical interaction between fractures as they deform [15]. In addition, the DD method has been demonstrated to calculate accurate stress distributions in the vicinity of fracture tips, which is necessary to model fracture propagation.

The model assumes a two-dimensional faulted and fractured domain, saturated with a single-phase fluid. The mechanical properties of the intact matrix rock are homogeneous. Deformations occur quasistatically, and the assumptions of linear elasticity apply.

The boundary element nature of the DD method requires only the fractures to be discretized. Following the approach described by Crouch and Starfield [15], a system of equations was developed that describes the opening and shear displacements of the fracture elements as a response to traction boundary conditions:

\[ \mathbf{t} = \mathbf{A} \mathbf{u}, \quad (6) \]

where \( \mathbf{t} \) is a vector of changes from the initial condition in normal and shear tractions along the fractures, \( \mathbf{A} \) is a matrix of DD interaction coefficients, and \( \mathbf{u} \) is a vector of normal and shear displacement discontinuities along the fractures. The interaction coefficients were calculated using the higher order DD method introduced by Shou and Crouch [16].

It should be noted that application of Eq. (6) is only necessary under certain local states of stress. Opening mode displacement discontinuities will only arise if the effective normal stress acting on a particular fracture plane is tensile. Shear mode displacement discontinuities will only occur if the shear traction acting on a particular plane overcomes its frictional resistance to slip. Therefore, during a simulation, the state of stress is continually evaluated at each fracture element to determine whether Eq. (6) applies, which can drastically reduce the total size of the system of equations that must be solved. Stress perturbations due to changes in fracture aperture when the effective stress is compressive are neglected, and shear displacements are assumed to be zero if the shear stress is below the frictional resistance to slip.

A sequential implicit strategy is used to solve the fully coupled system of equations [17, 18]. The primary variables involved are fracture pore pressure, matrix
pore pressure, opening displacement discontinuity, and shear displacement discontinuity. The residual equations involving pore pressure and opening displacement are grouped and solved simultaneously, and the residual equations for shear displacement are solved separately. The sequential strategy iterates between these two groups until all residual equations have converged to within a prescribed tolerance.

2.3. Permeability Evolution, Friction, and Fracture Propagation

A mandatory feature of any reservoir stimulation model is the ability to incorporate a mechanism for reservoir permeability to evolve due to the occurrence of failure of the rock through either shear or tension. In addition, permeability may be altered throughout the reservoir due to changes in the local state of stress.

In the present model, a distinction is made between fractures that are “open” and fractures that are “closed.” If a fracture element is bearing compression, that fracture is considered closed. The aperture of a closed fracture is a function of the effective normal stress and the amount of dilation due to shear slip [19]. In this case, the fracture aperture can simply be calculated with an empirical equation. If the effective stress acting is such that the walls of the fracture are in tension, the fracture is considered open. In this case, Eq. (6) must be used to solve for the opening displacement numerically. The fracture aperture for open elements is then the sum of the opening mode displacement discontinuity and the shear dilation due to slip. The transmissivity of each fracture element is a direct function of aperture, and is calculated according to the cubic law [20].

Friction evolution is extremely important for modeling reservoir stimulation processes, because the frictional strength of the fracture surfaces controls shear slip behavior. Shear slip can not only have the effect of altering permeability, but also can significantly perturb the local state of stress throughout the reservoir. Modeling friction evolution also has consequences for interpreting microseismic events and understanding induced seismicity.

The present model provides two approaches for modeling friction evolution. The first is a static – dynamic approach. In this method, a constant static coefficient of friction is used until the Mohr-Coulomb failure criterion is met, whereupon a dynamic coefficient of friction with a lower value is used. Once the sliding velocity of a fracture element recedes to a low value, the static friction coefficient is restored. The second approach uses a rate and state friction framework [21]. In this method, the coefficient of friction is a function of sliding velocity and sliding history (i.e., state). Rate and state friction is the leading theory for modeling earthquake nucleation and propagation. The differences between the two approaches are described in detail by McClure [1].

Fracture propagation is allowed to occur subject to the critical stress intensity factor criterion. The stress intensity factor at fracture tips is calculated as a function of the opening mode displacement discontinuity [22]:

$$K_t = \frac{G}{4\pi(1-\nu)} \left( \frac{2\pi}{a} \right)^{1/2} \Delta e,$$

where \( G \) is shear modulus, \( \nu \) is Poisson’s ratio, \( a \) is the half-length of the fracture tip element, and \( \Delta e \) is the opening mode displacement discontinuity at the fracture tip element. When the value of \( K_t \) reaches the critical stress intensity factor, \( K_C \), the fracture is allowed to propagate, and an additional fracture element is appended to the fracture. Once a new fracture element has nucleated, the appropriate matrix-fracture mass transfer term is activated in the mass balance equations (see Eqs. 1 and 2).

3. INJECTION INTO AN INFINITE-CONDUCTIVITY FRACTURE

The principal contribution of this work was to incorporate a framework for matrix-fracture mass transfer into an existing reservoir stimulation model. As described in Section 2.1, this was accomplished through application of the embedded fracture modeling approach. Equations (3) and (4) introduce additional approximations to the governing equations beyond the conventional finite volume discretization. The EFM framework has previously been verified to attain a suitable level of accuracy for a problem involving one-dimensional leakoff from a fracture into the surrounding rock [12]. That was a rather benign physical scenario, because the derivation of the fracture index (see Eq. (5)) involved assuming that flow near the fracture is indeed linear. In this section, we present a numerical experiment to verify the EFM mass transfer model for a slightly more complex scenario involving a transition from linear to radial flow.

3.1. Model Scenario and Analytical Solution

We considered injection into an infinite-conductivity fracture in an infinite domain. The reservoir behavior for this problem is linear flow at early times, followed by a transition toward radial flow. A closed-form solution exists for the transient pressure response at the well [23]:

$$p_D^w(t_D) = \frac{1}{2} \sqrt{\frac{\pi D}{6}} \left[ \text{erf} \left( \frac{0.134}{\sqrt{t_D}} \right) + \text{erf} \left( \frac{0.866}{\sqrt{t_D}} \right) \right] + 0.067 \text{Ei} \left( \frac{0.018}{t_D} \right) + 0.433 \text{Ei} \left( \frac{0.750}{t_D} \right).$$


The dimensionless variables are defined as:

\[ p_D^w = \frac{2\pi k''H}{q'' \mu} \left( p^w - p_0 \right), \]  

and

\[ t_D = \frac{\mu \phi c_t}{k'' x_f} t. \]

All previously undefined variables are listed in the Notation section.

The model parameters used in the simulations are given in Table 1. Four levels of matrix grid refinement were considered. The number of control volumes used to discretize the fracture domain remained constant for all of the simulations. No geomechanical effects were considered. Fluid was injected at a constant rate for a period of 48 hours. No wellbore storage effects were considered.

3.2. Numerical Results

Figure 2 illustrates the pressure response observed at the well over the duration of injection for the various levels of grid refinement compared directly to Eq. (8). The finest level of grid refinement appears to match the analytical solution very well. For coarser levels of grid refinement, the solutions deviate from the true solution at early times, but converge towards the true solution as time proceeds. The solutions appear convergent upon grid refinement.

The pressure derivative for the case with the highest level of grid refinement is also plotted in Fig. 2. The \( \frac{1}{2} \) slope of both the pressure response and the pressure derivative at early times is indicative of the linear flow regime associated with flow near an infinite-conductivity fracture. The transition toward radial flow occurred at roughly two hours and is captured well. This result is encouraging because it indicates that it is possible to use the EFM approximation to consider cases where flow may not be strictly linear.

The relative error over the duration of the injection period is illustrated in Fig. 3. It was observed that the error never exceeded 2.5% for each of the levels of discretization. At early times, the errors in wellbore pressure are positive, indicating that the amount of fluid leakoff is underestimated. This result is consistent with observations from a previous study [12]. However, at late times, the errors in wellbore pressure are tending towards negative values. This is an indication that the amount of fluid leakoff is overestimated. The errors appear to be undulating, which may suggest that the leakoff errors could be self-correcting, but this observation is not conclusive given the time span covered by these simulations. The self-correcting nature of the errors would be physically intuitive, because the leakoff is driven by pressure gradients.

4. HYDRAULIC FRACTURE PROPAGATION

The main motivation for extending the model introduced by McClure [1] to include the EFM framework is to allow for matrix-fracture mass transfer in settings where the fracture system is dynamic, because the method does not require a conforming grid. In this section, we demonstrate the EFM framework’s adaptive nature by showing the application of the model to a fracture propagation problem.

4.1. Model Scenario and Analytical Solution

We considered the propagation of a vertical two-wing hydraulic fracture in a homogeneous domain with no preexisting natural fractures. The model assumes plane strain in the vertical direction, and so we will compare our simulation results with the KGD analytical solution for this problem.

Gidley et al. [24] provide a closed form solution that describes the fracture length as a function of time for a constant injection rate and assuming no leakoff:

\[ x_f = 0.679 \left[ \frac{G_i^3}{(1-v) \mu h_f^3} \right]^{1/6} t^{2/3}, \]

where \( x_f \) is the fracture half-length, \( i \) is half of the total volumetric injection rate (i.e., the flow rate entering one of the fracture wings), and \( h_f \) is the height of the fracture.

For the case where leakoff is considered, Valko and Economides [25] performed a material balance that yielded a non-linear function for fracture length (neglecting spurt-loss):

\[ x_f = \frac{\bar{e} \iota}{4\pi C_L h_f} \left[ \exp\left(\beta^2\right) \text{erfc}\left(\beta\right) + \frac{2\beta}{\sqrt{\pi}} - 1 \right] \]

where \( \bar{e} \) is the average aperture of the fracture and \( C_L \) is the leakoff coefficient. The time variable is included in the following parameter:

\[ \beta = \frac{2C_L \sqrt{\pi t}}{\bar{e}}. \]

The nonlinearity arises because the average fracture aperture at any time depends on the length of the fracture. Gidley et al. [24] suggest that the fracture aperture at the wellbore, \( e^w \), is:
Table 1. Model parameters for study of injection into an infinite-conductivity fracture.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^w$</td>
<td>0.001</td>
<td>m³ · s⁻¹</td>
</tr>
<tr>
<td>$x_f$</td>
<td>50</td>
<td>m</td>
</tr>
<tr>
<td>$h_f$</td>
<td>1</td>
<td>m</td>
</tr>
<tr>
<td>$e$</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td>$p_0$</td>
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<td>MPa</td>
</tr>
<tr>
<td>$k_m$</td>
<td>$20 \times 10^{-15}$</td>
<td>m²</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.001</td>
<td>Pa · s</td>
</tr>
<tr>
<td>$c_t$</td>
<td>$8.8 \times 10^{-10}$</td>
<td>Pa⁻¹</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.2</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 2. Pressure response at the well over the 48 hour injection period for different levels of grid refinement. For the sake of clarity, the case with 40401 matrix control volumes was excluded from this plot. The pressure derivative for the highest level of refinement is shown as blue circles. Of particular interest is the HFM simulator's ability to capture the transition to radial flow.

Figure 3. Illustration of the relative error for each of the four levels of matrix grid refinement. The relative error was calculated in dimensionless space with reference to the analytical solution, and normalized by the change in pressure at the end of the simulation. The errors never exceeded 2.5% over the duration of the simulations, indicating the EFM approximation is able to resolve behavior other than purely linear flow (e.g., radial flow in this case).
\[ e^* = 2.27 \left( \frac{\mu \xi_f^2 (1 - \nu)}{Gh_f} \right)^{1/4}. \] (14)

For the KGD fracture geometry, the average value of fracture aperture is:

\[ \bar{e} = \frac{\pi}{4} e^*. \] (15)

If the leakoff coefficient is known, Eqs. (13) – (15) can be substituted into Eq. (12), so that Eq. (12) becomes only a function of time and fracture length. A suitable nonlinear solver can then be used in a time marching procedure to calculate fracture length as a function of time. Note that Eq. (12) is a general expression, whereas Eq. (11) is only valid for the specific form of the aperture at the wellbore defined in Eq. (14).

In our study, we considered leakoff to be slightly compressible flow driven purely by diffusion into the rock matrix. Under this assumption, the leakoff coefficient is [26]:

\[ C_t = \left[ \frac{k^w \phi_e}{\pi \mu} \right] (p' - p_0). \] (16)

The model parameters for the numerical example are given in Table 2. Fluid was injected at a constant rate for a total of 30 minutes. The fracture was assumed to have a fixed height in the vertical direction. The magnitude of the minimum principal stress was five megapascals above the initial reservoir pressure, so the relatively high pressure in the fracture necessary to drive the hydraulic fracture encouraged leakoff to occur. In order to determine the leakoff coefficient used to calculate the analytical solution to the problem (see Eq. (12)), the pressure drop was assumed to be \( \Delta p = p' - p_0 = 5.2 \) MPa. In order to investigate the effects of the matrix discretization on the fracture propagation behavior, three different levels of grid refinement were tested. For reference, the case where no leakoff occurs was also considered.

### 4.2. Numerical Results

The simulation results are compared directly to the analytical solution in Fig. 4, and Tables 3 and 4 list summaries of the results at the end of pumping. For the case where no leakoff occurred, it was observed that the numerical solution slightly underestimated the fracture length for any given time compared to Eq. (11). At the time pumping stopped, the numerical results underestimated the total fracture length by 4.5%.

It is worthwhile to note that the literature provides several alternatives to Eq. (11). The differences arise due to assumptions that are made in the derivation of Eq. (14). The different approaches yield expressions that are similar in form to Eq. (11), but differ by the constant that appears out front. For example, Valko and Economides [25] suggest that the constant is 0.605 instead of 0.679. The difference in fracture length predicted by using these two constants is on the order of 10%. Our numerical solution fell within this range.

The effect of fluid leakoff is that some of the injected fluid volume is lost to the formation and is not useful for extending the hydraulic fracture. Qualitatively, it is observed in Fig. 4 that the numerical solutions follow the behavior of the analytical solution. At the time pumping stopped, the analytical solutions predict that the fracture is roughly 21% shorter when leakoff is considered. Comparing the numerical solutions to each other, the hydraulic fracture was predicted to be 19.8% shorter when leakoff was considered (for the highest level of grid refinement). This result is extremely encouraging because it verifies that the EFM approximation is valid even when the fracture system grows over time.

### 5. Concluding Remarks

In this paper, we have presented the formulation for a reservoir simulator based on an embedded fracture modeling framework. The physical processes considered were mass transfer between fractures and surrounding matrix rock, mechanical deformation of fractures and surrounding matrix rock, permeability evolution, friction evolution, and fracture propagation. The EFM approach was adopted in order to avoid problematic numerical issues associated with matrix discretizations that conform to fractures. Two numerical experiments were performed in order to demonstrate the model’s utility and to verify its accuracy.

The first study considered injection into an infinite-conductivity fracture. The most intriguing result that emerged from this study was that even though the fracture index is derived assuming linear flow near the fracture, the expected transition towards radial flow was observed. In concept, this is similar to the common reservoir simulation practice of applying the Peaceman well index in situation in which the flow is not purely radial.

The second study considered propagation of a single two-wing vertical fracture. The numerical model matched the KGD analytical solutions within a tolerable range. This example proved that the EFM provides a simple, yet effective framework for problems in which the fracture domain evolves over time.

The EFM method deserves further analysis of its numerical properties. A future topic of research will be to compare EFM directly with a traditional DFM simulator for more complex scenarios involving fracture propagation, because a tradeoff between accuracy and performance certainly exists for the two approaches.
Table 2. Model parameters for mode-I hydraulic fracture propagation study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Parameter</th>
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<td>$\phi$</td>
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<td>MPa·m$^{1/2}$</td>
</tr>
<tr>
<td>$\mu$</td>
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<td>Pa·s</td>
<td>$G$</td>
<td>15</td>
<td>GPa</td>
</tr>
<tr>
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<td>Pa$^{-1}$</td>
<td>$\nu$</td>
<td>0.25</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 4. Comparison of KGD analytical solutions with the numerical model. The numerical model yielded results that were within 5% of the analytical solutions for both the leakoff and zero leakoff cases. For the cases where leakoff was allowed to occur, the grid refinement study suggested that the EFM approach is convergent upon refinement and is applicable for cases where the fracture system is growing over time.

Table 3. Fracture half-length at the time pumping stopped.

<table>
<thead>
<tr>
<th></th>
<th>No Leakoff</th>
<th>Leakoff</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KGD</td>
<td>Numerical</td>
</tr>
<tr>
<td>$x_f$ [m]</td>
<td>370.0</td>
<td>353.2</td>
</tr>
<tr>
<td>Rel. Error [%]</td>
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<td>-4.5</td>
</tr>
</tbody>
</table>

Table 4. Reduction in fracture half-length due to leakoff effect. The values listed for fine, medium, and coarse levels of discretization were compared to the numerical solution for no leakoff.

<table>
<thead>
<tr>
<th>Model</th>
<th>Reduction in $X_f$ Due to Leakoff [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>KGD</td>
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</tr>
<tr>
<td>Fine</td>
<td>19.8</td>
</tr>
<tr>
<td>Medium</td>
<td>18.8</td>
</tr>
<tr>
<td>Coarse</td>
<td>15.1</td>
</tr>
</tbody>
</table>
NOTATION

\( a \) fracture element half-length [m]
\( A \) surface area of one face of fracture control volume [m²]
\( A' \) total surface area of fracture control volume [m²]
\( A_{di} \) displacement discontinuity interaction coefficient matrix [Pa ∙ m⁻¹]
\( c_i \) total compressibility [Pa⁻¹]
\( C_L \) leakoff coefficient [m ∙ s⁻¹/²]
\( \langle d \rangle \) normal distance from fracture averaged over a matrix control volume [m]
\( e \) fracture hydraulic aperture [m]
\( \bar{e} \) average aperture of hydraulic fracture [m]
\( e^* \) aperture at the wellbore of hydraulic fracture [m]
\( E \) fracture void aperture [m]
\( G \) shear modulus [Pa]
\( h_f \) fracture height [m]
\( H \) formation thickness [m]
\( i \) one half of the total volumetric pumping rate [m³ ∙ s⁻¹]
\( k' \) fracture permeability [m²]
\( k_m \) matrix permeability [m²]
\( K_I \) mode-I stress intensity factor [Pa ∙ m¹/²]
\( K_{IC} \) critical mode-I stress intensity factor [Pa ∙ m¹/²]
\( p' \) pressure in the fracture domain [Pa]
\( p_m \) pressure in the matrix domain [Pa]
\( p_w \) wellbore pressure [Pa]
\( p_0 \) dimensionless wellbore pressure [-]
\( p_b \) initial reservoir pressure [Pa]
\( q' \) volumetric fluid injection rate [m³ ∙ s⁻¹]
\( \tilde{q}_m \) normalized fracture-matrix mass transfer [kg ∙ s⁻¹ ∙ m³⁻¹]
\( \tilde{q}^*_m \) normalized matrix-fracture mass transfer [kg ∙ s⁻¹ ∙ m²⁻¹]
\( \tilde{q}^*_{fm} \) normalized fracture-matrix mass transfer [kg ∙ s⁻¹ ∙ m³⁻¹]
\( \tilde{q}^{nf} \) normalized volumetric source [s⁻¹]
\( \tilde{q}^{nw} \) normalized volumetric source [s⁻¹]
\( t \) time [s]
\( t_D \) dimensionless time [-]
\( t \) traction vector [Pa]

REFERENCES


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