Seismogenic index and magnitude probability of earthquakes induced during reservoir fluid stimulations

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An important characteristic of seismicity is the distribution and magnitude of earthquakes. Fluid injection in rocks, aimed to create enhanced geothermal systems (EGS), can sometimes produce significant seismic events (e.g., Majer et al., 2007). This is rarely the case in hydraulic fracturing of hydrocarbon reservoirs. However, in any case the behavior of the seismicity triggering in space and in time is controlled by the process of stress relaxation and pore-pressure perturbation that was initially created at the injection source. This relaxation process can be approximated by pressure diffusion (possibly a nonlinear one) in the pore fluid of rocks (e.g., Shapiro and Dinske, 2009). At some locations the tectonic stress in the Earth’s crust is close to a critical stress, causing brittle failure of rocks. Increasing fluid pressure in such a reservoir causes pressure in the connected pore and fracture space of rocks to increase. Such an increase in the pore pressure consequentially causes a decrease of the effective normal stress. This leads to sliding along pre-existing, favorably oriented, subcritical cracks.

Recently we have found that under rather general conditions the number of fluid-injection-induced earthquakes with a magnitude larger than a given one increases approximately proportionally to the injected fluid volume (Shapiro et al., 2007; Shapiro and Dinske). Here we show that, using the seismicity rate of induced events and the fluid-injection rate, we can derive a parameter (the seismogenic index) that quantifies the seismotectonic state at an injection location. This index is independent of injection parameters and depends only on tectonic features. The seismogenic index can be used to quantitatively compare different locations of a possible fluid injection (e.g., for a geothermal or a CO₂ injection) in terms of the potential risk of inducing an event of a magnitude larger than a given one. Along with injection parameters, the knowledge of the seismogenic index allows us to estimate the occurrence probability of a given number of such events during an injection period. In the following, we will work with a very simple model of fluid-induced seismicity. In spite of the simplicity of the model, it seems to describe a broad spectrum of applications. We consider a point-like injection source in an infinite and homogeneous medium. Due to a fluid injection and the consequent process of pressure relaxation, the pore pressure (p) will change throughout the pore space. We assume that a random set of pre-existing cracks (defects) is statistically homogeneously distributed in the medium and is characterized by the volume concentration \( N \). For simplicity we assume that the cracks do not mutually interact. Each crack is characterized by an individual critical value (\( C \)) which is the pore pressure necessary to create displacement along such a defect. This critical pressure \( C \) is randomly distributed on a set of pre-existing cracks. Statistical properties of \( C \) are assumed to be independent of spatial locations. If at a given point \( r \) a pre-existing crack, the pore pressure \( p(t,r) \) increases with time and equals \( C(r) \) at time \( t_0 \), then this point will be considered the hypocenter of an earthquake. For simplicity, we assume that no earthquake will be possible at this point again. This is equivalent to assuming that the stress corrosion, tectonic load, tectonic deformation and other phenomena leading to a recharging of critical cracks are much slower than the process of the pore-pressure relaxation. Recent work of Rothert and Shapiro (2007) indicates that in the first approximation \( C \) is uniformly distributed between its minimal and maximal values, \( C_{\text{min}} \) and \( C_{\text{max}} \). These values are of the order of \( 10^2 \) and \( 10^6 \) Pa, respectively. \( C_{\text{max}} \) is usually larger than the injection-caused pressure perturbation (excluding maybe a small volume around the source).

These assumptions lead to the statement that the event probability is proportional to the pore-pressure perturbation. This statement is valid in the case of increasing or constant injection pressure. The number of events induced from start of injection until the current time moment \( t \) is given by a spatial integral of pressure perturbations. Under rather general conditions including a strongly nonlinear interaction between an injected fluid with a rock, such an integral can be estimated from the mass conservation principle, a principle valid independently of any injection impact onto the rock permeability (i.e., it is not important if a hydraulic fracture has been created or not). Application of this principle yields the following relation for the cumulative number of induced seismic events:

\[
N_{\text{ev}}(t) = N Q(t) / (C_{\text{max}} S)
\]

where \( N \) is a concentration of preexisting cracks, \( S \) is a poroelastic uniaxial storage coefficient, \( Q(t) \) is a cumulative injected fluid volume. The quantity \( F_I = C_{\text{max}} / N \) depends on tectonic activity of the injection region only. Shapiro et al. refer to this quantity as a “tectonic potential.” We see that the number of events induced is proportional to the fluid volume injected. Note that pore-pressure perturbation is assumed constant or increasing.

To compute the number of events with magnitude larger than a given one, we assume that the probability, \( W \geq M \), of a given event to have a magnitude larger than \( M \) obeys a Gutenberg-Richter statistic. A Gutenberg-Richter statistic of induced seismicity might be, e.g., a consequence of a power-law-type size distribution of pre-existing cracks. The Gutenberg-Richter magnitude scale means that the probability \( W \geq M \) of events with the magnitude larger than \( M \) is given by

\[
\log W \geq M = a - b M,
\]

where \( a \) and \( b \) are seismotectonic constants. Thus, for the expected number of events with magni-
Seismic activity larger than a given one, we obtain

$$\log N \geq M(t) = \log Qc(t) - bM + a - \log(FtS) \quad (2)$$

In the following, we will use this equation to introduce the seismogenic index, characterizing seismotectonic activity of a fluid-injection site, and to formulate the occurrence probability of a given number of events with magnitude larger than a given one.

**Seismogenic index**

Figures 1 and 2 demonstrate the good correlation between Equation 2 and the magnitude distribution at the Basel experiment (see below for a description of the Basel data). We see an approximately linear dependence between $N \geq M(t)$ and the cumulative injected volume. On the corresponding log-log plot (Figure 2), a straight line with proportionality coefficients close to 1 would provide a good fit to the data. Note that this slope is not the $b$ value. In terms of Figure 2, the $b$ value controls the separation of points corresponding to different magnitude ranges rather than slopes.

Let us rewrite Equation 2 in the following form:

$$\log N \geq M(t) - \log Qc(t) + bM = a - \log(FtS) \quad (3)$$

The quantity on the left side of this equation is an experimentally measurable one. It depends on the injection parameters and on the induced seismicity. We are concentrating now on the quantity $\Sigma$ defined by the right side of this equation:

$$\Sigma = a - \log(FtS) \quad (4)$$

This quantity is independent of injection time and of any other injection characteristics. It is completely defined by seismotectonic features of a given location. We will call it a “seismogenic index.” The larger this index, the larger the probability of significant magnitude event. Once the seismogenic index is measured for an injection site, it can be used with the $b$ value of the Gutenberg-Richter distribution to predict the number of events within a specific magnitude range using the equation:

$$\log N \geq M_1 = \log Qc_1(t) - bM_1 + \Sigma$$

where $Qc_1$ and $M_1$ are cumulative injected volume and an arbitrary event magnitude for a new injection experiment at the same site. Of course, such a prediction will be valid only, if $b$ and $\Sigma$ are not changing significantly with time.

The seismogenic index is a convenient quantity for a quantitative comparison of seismotectonic activity at different locations. Here we compute the seismogenic index at several borehole injection locations, both geothermal sites and sites of hydraulic fracturing of hydrocarbon reservoirs. For our discussion, we use time periods in these experiments that correspond to nondecreasing injection rates. We have tried to restrict our analysis to magnitude ranges that minimize errors in the observation systems, event registration and event processing. We also attempted to use moment magnitudes. Where moment magnitudes were not given, our estimates of the seismogenic index are biased by the order of difference.
between the local magnitudes and the moment magnitudes in corresponding magnitude ranges. To our knowledge, this bias for all locations shown here is insignificant.

We consider three geothermal locations: Cooper Basin, Basel, and Ogachi. We include also three nongeothermal sites: Paradox Valley, Cotton Valley, and Barnett Shale. The last two locations correspond to hydraulic fracturing of gas reservoirs.

Firstly, we compare the data related to enhanced geothermal projects in Australia at the Cooper Basin, in Switzerland at Basel, and in Japan at Ogachi, respectively. Cooper Basin is in South Australia. We consider here the microseismicity induced in the borehole Habanero-1 (Soma et al., 2004), which was drilled to a depth of 4421 m. The main outflow zone was at a natural fracture in granite at 4254 m. The main injection period lasted approximately 11 days. In this time about 14,000 m³ of water were pumped into the borehole and more than 2800 events were induced.

Basel is in the Upper Rhine Graben (Dyer et al., 2008). Borehole Basel-1 was drilled to 4800 m. The open hole section (also in granite) corresponds approximately to the last 400 m of the borehole. This borehole had a short main outflow zone close in the upper part of the open-hole section. During the period of continuous injection, about 11,000 m³ of water were pumped into the borehole and about 2300 events were located.

During an experiment at the Ogachi geothermal site in 1991 (Kaieda et al. 1993), more than 10,000 m³ of water were injected at a depth of 1000 m into granodiorite.

At Paradox Valley, the injection was carried out in several phases between 1991 and 2004 in order to reduce salinity in the Colorado River (Ake et al., 2005). The brine was injected into a fractured limestone formation at a depth of more than 4 km. About 4000 events with magnitude larger than −0.5 were induced.

The two last case studies are hydraulic fracturing injections into tight-gas reservoirs in the Cotton Valley sandstones and Barnett Shale.

We consider two hydraulic fracturing experiments in the Cotton Valley gas field of Texas. The gas reservoir is a low-permeability, interbedded sequence of sands and shale (Rutledge and Phillips, 2003). The hydraulic fracturing was conducted in the depths of 2757–2838 m and 2615–2696 m. About 990 and 650 microearthquakes were located.

The other hydraulic fracture example comes from microseismic data collected from the Barnett Shale (Maxwell et al., 2006). They were obtained by hydraulic fracturing from a vertical borehole in the Lower Barnett Shale of the Fort Worth Basin.

We estimate the seismogenic index $\Sigma$ for all locations mentioned above using Equation 3 (Figure 3). They demonstrate reasonable stability as they do not fluctuate as function of time. Statistical errors and temporal fluctuations of the estimates are small and in the range of ±0.5. We expect that the bias caused by differences between local and moment magnitudes is of a similar size to the statistical errors. This is indirectly confirmed by comparison of different magnitude studies for same locations (e.g., Mukuhira et al., 2008, and Deichmann and Giardini, 2009, for the Basel experiment).

We observe the following general tendency. The seismogenic index at the hydraulic fracturing sites is smaller than at geothermal sites. In our examples of hydrocarbon reservoirs, it is less than −4 and larger than −3 at the geothermal sites. Significantly, a higher seismogenic index can be estimated for the Cooper Basin data set. It is of the order of $-1.2 \pm 0.9$. The largest index is obtained for the Basel data set: 0.1 ± 0.7.

Another interesting observation is that at Cotton Valley, the A and B gel fracturing stages are characterized by a significantly larger $\Sigma$ than the water-fracturing stage. The water-fracturing stages of the Barnett Shale and of the Cotton Valley have similar $\Sigma$. Possibly this is an indication that the physics and chemistry of treatment fluids can influence the seismogenic index. On the other hand, the gel and water stages of the Cotton Valley hydraulic fracturing were at two different boreholes. The microseismicity of the gel-fracturing stages (especially of the b stage) indicates a presence of large-scale natural fractures in the corresponding part of the reservoir. This could be another explanation of the larger $\Sigma$ of the gel stages.

**Occurrence probability of events with given magnitudes**

A higher seismogenic index leads to a higher probability of significant events. Let us first assume a constant injection rate, $Q(t)$. Then, the cumulative event number will be $N_{ev(t)} = vt$, where $v = NQ/r$ is a constant temporal event rate. Our model and assumptions described above imply that the events occur independently. In other words, in our model the
induced seismicity is assumed to follow a Poisson distribution like, e.g., a radioactive decay. A sequence of events with magnitude larger than $M$ is also a Poisson process. In the case of an arbitrary nondecreasing injection rate, the induced seismicity is an inhomogeneous Poisson process with a temporal event rate that is not constant.

Using the Poisson distribution, we can compute the occurrence probability of $n$ events with magnitude larger than $M$ in a specified time interval. Of particular practical importance is a probability of the absence of an event with a magnitude larger than a given $M$ in the time interval from 0 (i.e., a start of an injection) until $t$:

$$P(0,M,t) = \exp(-N \geq M(t)) = \exp(-Q_c(t)10^{\frac{M}{b}}) \quad (5)$$

where $N \geq M(t)$ is given by Equation 1 and in the last part of the equation we have used the Gutenberg-Richter magnitude distribution.

Let us assume that we want to exclude occurrence of events with magnitude larger than $M$ with a probability $99\%$. This means that $P(0,M,t) = 0.99$. Substituting this into Equation 5 allows us to compute a tolerable maximal cumulative injected volume or, by a given injection rate, the maximal possible injection time.

For a numerical illustration, we consider an injection experiment with parameters close to the Basel experiment. We assume that the seismogenic index $\Sigma$ is 0.25. We also assume that during 5.5 days of the stimulation, 11,570 m$^3$ of fluid were injected into the rock. We assume further that the $b$ value is approximately 1.5. What is the probability that, during such an injection, events with magnitude larger than $M = 2.5$ will occur? This probability is given by substituting the injection parameters into Equation 5, computing the value of $P(0,M,t)$, computing then the value of $1 - P(0,M,t)$ and multiplying the last value by 100%. We obtain approximately 97%. The occurrence probability of events quickly decreases with increasing magnitudes (see Figure 4). For example, the probability that events with magnitude larger than 3.5 will occur is approximately 11%.

In the Basel injection, the largest event with the reported local magnitude of 3.4 occurred approximately 5 hours after the 6 days of continuous injection stopped (Häring at al., 2008). Strictly speaking, this event happened outside of the time interval described by our model. However, our Figure 4 and the probability values given above seem realistic in the light of this event.

**Conclusions**

We have introduced a quantity, seismogenic index that characterizes the level of seismic activity one should expect from injecting fluid into rocks. Six case studies show that the largest seismogenic index characterizes the Basel geothermal site and the smallest are at hydrocarbon reservoirs. This observation shows that microseismic monitoring of hydrocarbon reservoirs requires, as a rule, a higher level of sensitivity than such monitoring of geothermal systems.

It has already been observed that injections at geothermal sites are characterized by lower $b$ values than injections at hydrocarbon reservoirs (Shapiro and Dinske). From this study, it becomes clear that geothermal locations are also characterized by a higher seismogenic index. These two factors explain the observation that injections at geothermal reservoirs have a tendency to sometimes induce significant events but do not at hydrocarbon reservoirs. This tendency is additionally enhanced by a relatively small injection volumes used in hydraulic fracturing of hydrocarbon reservoirs compared to stimulations of geothermal systems.

Fluid-induced seismicity follows Poisson statistics. The seismogenic index and fluid injection rate are two key controlling parameters. We provide explicit rules for estimation of occurrence probability for induced events of given magnitudes. All the results we discussed apply to a time period of active fluid injection.

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