Evidence for a transient hydromechanical and frictional faulting response during the 2011 $M_w$ 5.6 Prague, Oklahoma earthquake sequence

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Abstract

Mechanisms for the delayed triggering between the $M_w$ 4.8 foreshock and $M_w$ 5.6 main shock of the 2011 earthquake sequence near Prague, Oklahoma, USA, were investigated using a coupled fluid flow and fault mechanics numerical model. Because the stress orientations, stress magnitudes, fault geometry, and earthquake source mechanisms at the Prague site have been well characterized by previous studies, this particular earthquake sequence offered an opportunity to explore the range of physical processes and in situ fault properties that might be consistent with the 20 h delayed triggering effect observed at the site. Our numerical experiments suggest that an initial undrained response resulting from elastic stress transfer from the foreshock followed by transient fluid flow along the fault may have contributed to the earthquake nucleation process. The results of the numerical experiments were used to constrain fault compliance and fault transmissivity for the fault that hosted the $M_w$ 5.6 event. Relatively compliant behavior in response to changes in normal stress, corresponding to Skempton pore pressure coefficients near 1, was consistent with the field observations. Fault transmissivity was estimated to range from $10^{-18}$ to $10^{-15}$ m$^3$. This study has implications for understanding hydraulic properties, frictional properties, and faulting behavior of basement faults in Oklahoma that are large enough to host damaging earthquakes.

1. Introduction

On 5 November 2011, a $M_w$ 4.8 earthquake (Event A) occurred near the town of Prague, Oklahoma, USA. The aftershock sequence and regional moment tensor indicated that this event occurred along a previously mapped portion of the Wilzetta Fault [Keranen et al., 2013; McNamara et al., 2015]. Based on recent measurements of the stress orientations and stress magnitudes in the area [Walsh and Zoback, 2016], this particular fault segment (Fault A) was oriented at roughly 50° from the direction of maximum principal stress. On the morning of 6 November 2011, roughly 20 h after Event A, a $M_w$ 5.6 earthquake (Event B) occurred in very close proximity to the prior event. Detailed analyses of the aftershock sequence and regional moment tensor for Event B indicated that the splay branch (Fault B) off the main Wilzetta Fault that hosted this event was optimally oriented for shear failure. Keranen et al. [2013] and Sumy et al. [2014] suggested that Event B was triggered by the elastic stress transfer caused by Event A.

In this study, we investigated possible mechanisms for the 1 day delayed triggering between Events A and B. The conceptual model we tested involved a strong hydromechanical coupling in which changes in the state of stress along a fault can induce pressure changes and fluid flow within the fault zone. We hypothesized that the delayed triggering of Event B would have been influenced by fluid pressure diffusion along the fault following an initial undrained loading response to Event A as well as a time-dependent impact on fault friction caused by state evolution.

Because the geometry of Faults A and B, the rupture dimension of Event A, and the stress state in the area were reasonably well characterized based on previous studies, this particular earthquake sequence provided an opportunity to learn about the in situ properties of basement faults in Oklahoma that are capable of hosting relatively large earthquakes. Ultimately, our goal was to constrain hydraulic and frictional properties of Fault B by exploring the range of scenarios that could explain the observation of the delayed triggering.

It has been recognized that gaining an improved understanding of the interactions between faulting processes and fluid flow along faults will have important implications for analyzing hazard related to injection-induced seismicity [Ellsworth, 2013; McGarr, 2014; McGarr et al., 2015]. For the first time,
injection-induced earthquakes have been included in an official 1 year seismic hazard forecast for the central and eastern United States [Petersen et al., 2016]. However, the seismic hazard model did not relate any injection well operational parameters, such as injection pressure or injection rate, to changes in seismicity. Physics-based models capable of assessing seismic hazard related to induced seismicity, such as those described by Norbeck and Horne [2015b] and Király-Proag et al. [2016], require detailed information about the hydraulic and mechanical properties of the faults that exist in the model. It remains difficult to use traditional reservoir engineering approaches, for example, pressure transient analysis, to measure the in situ properties of basement faults that would be necessary to inform physics-based models of induced seismicity. Extending behavior observed in the laboratory for fractured and faulted rocks to large fault structures can be questionable. As a practical alternative, we sought to identify and interpret signatures of hydromechanical behavior that would be useful for inferring properties of the basement faults involved in the Prague earthquake sequence.

The remainder of this paper is organized as follows. In section 2 we present the hydrogeologic and geomechanical conceptual model used in our study. In section 3, we describe our fluid flow and fault mechanics numerical model. An overview of the theoretical background relevant to analysis of transient flow along faults and rate-and-state friction evolution is provided in section 4. The results of the numerical experiments are provided in section 5. Finally, we discuss the implications of our results for understanding faulting in Oklahoma in section 6 and present several concluding remarks in section 7.

2. Hydromechanical Conceptual Model

During an earthquake rupture, stresses are transferred through the material surrounding the fault. At time scales relevant to the propagation of the earthquake rupture along the fault (on the order of seconds), the extent of the stress transfer is mediated by the elastic wave speed. After an earthquake rupture has arrested, dynamic effects no longer play a role, and the stress change is effectively locked in. A common approach for analyzing the effects of elastic stress transfer induced by large earthquakes on subsequent earthquake activity is to calculate the Coulomb stress change resolved on the fault plane of interest as [Pollard and Fletcher, 2005]

\[
\Delta \sigma_C = \Delta \tau - f (\Delta \sigma - \Delta p)
\]

where \( \tau \) is shear stress, \( \sigma \) is normal stress, \( p \) is fluid pressure within the fault zone, and \( f \) is the coefficient of friction. Compressive normal stresses are taken as positive in this sign convention. A positive \( \Delta \sigma_C \) indicates that the new stress state is more favorable for shear failure.

Sumy et al. [2014] performed a Coulomb stress analysis of the Prague earthquake sequence which suggested that the magnitude of the stress perturbations caused by Event A were sufficient to trigger Event B. However, the 1 day delayed triggering implies that some time-dependent process controlled behavior during the sequence. Poroelastic effects, in which stress changes induce pressure changes and fluid flow, have been proposed as a possible physical mechanism for delayed aftershock triggering [Cocco and Rice, 2002; Nur and Booker, 1972; Roeloffs, 1996]. As a practical example, Bosl and Nur [2002] analyzed the spatial and temporal distributions of aftershocks during the 1992 Landers sequence and illustrated a good correlation between the occurrence of aftershocks and zones of increasing Coulomb stress. Alternatively, some models of fault friction, such as the rate-and-state model, include temporal components that can reproduce realistic aftershock delay behavior [Dieterich, 1992, 1994; Helmstetter and Shaw, 2008; Segall, 2010]. Segall and Lu [2015] investigated the combined effects of poroelastic stressing and rate-and-state earthquake nucleation in the framework of a continuum-based semianalytical model and highlighted the interaction between transient fluid flow and friction evolution.

We developed a conceptual model of the Prague earthquake sequence based on interpretations of mapped faults, aftershock sequences, focal mechanisms, and measurements of the regional state of stress. We investigated the relative influence of hydromechanical and friction evolution effects by performing several numerical experiments using a reservoir model that calculated the coupled interaction between fluid flow, fault deformation, and earthquake rupture within a rate-and-state friction framework. The model geometry is illustrated in Figure 1. Here we provide an overview of the sources of information used to construct the model.
Figure 1. Illustration of the two-dimensional fault model geometry. Fault A represents a portion of the Wilzetta Fault that hosted the foreshock, and Fault B represents the splay branch that hosted the main shock. The stress regime in this area of Oklahoma is strike-slip with the maximum horizontal stress oriented at N85°E. The blue and red stars are the modeled epicenters of Event A and Event B, respectively.

2.1. Hydrogeology and Stress State Near Prague
Throughout most of Oklahoma, Precambrian basement rock is overlain directly by a sedimentary aquifer called the Arbuckle formation. Nelson et al. [2015] demonstrated how the fluid pressure in aquifers in the midcontinent of the United States is controlled by the elevation at which each unit outcrops. In the area near Prague, Oklahoma, Nelson et al. [2015] estimated that the Arbuckle exists in a state of natural underpressure with an equivalent fluid pressure gradient of approximately 9 MPa/km. We assumed that Faults A and B were connected hydraulically with the Arbuckle aquifer because many of the aftershocks from both events were located in the sedimentary sections above the basement [Keranen et al., 2013; McNamara et al., 2015]. Alt and Zoback [2014] analyzed wellbore image data for many oil and gas wells in Oklahoma to determine stress orientations and found that the orientation of the maximum horizontal stress in the area near Prague was roughly N85°E. Walsh and Zoback [2016] performed stress inversions using a focal mechanism analysis of 15 earthquakes near Prague to determine stress orientations and magnitudes in the area. That analysis assumed an overburden gradient of $\gamma_{\sigma_z} = 25$ MPa/km, a fluid pressure gradient of $\gamma_p = 9$ MPa/km, and a static friction coefficient of $f_s = 0.7$. The stress regime was found to be strike-slip, and the orientation of the maximum principal stress was found to be N83°E, in close agreement with Alt and Zoback [2014]. The minimum and maximum horizontal stress gradients were estimated as $\gamma_{\sigma_h} = 15$ MPa/km and $\gamma_{\sigma_H} = 30$ MPa/km, respectively.

2.2. Fault Structure Geometry
The fault structure geometry for the model used in our numerical experiments (see Figure 1) was based on interpretations of seismic data performed by Keranen et al. [2013], Sumy et al. [2014], Sun and Hartzell [2014], and McNamara et al. [2015]. Focal mechanisms from the major earthquakes and many smaller aftershocks indicated that for both Events A and B, slip occurred on near-vertical faults with a predominantly strike-slip sense of motion [McNamara et al., 2015; NCEDC, 2014]. The focal mechanisms for the earthquakes each had a nodal plane orientation that was consistent with the distribution of aftershock locations. The hypocentral depths of Events A and B were recorded as 4.0 km and 7.0 km, respectively, although McNamara et al. [2015] suggested that these depth estimations have large errors associated with them. The inversion performed by Sun and Hartzell [2014] suggested that slip on Fault B nucleated at a hypocentral depth of 5 km and was confined initially between depths of 4 and 6 km. In their analysis, Sumy et al. [2014] estimated the rupture dimension for Event A as 2.8 km long by 2.9 km deep and for Event B as 8.3 km long by 5.4 km deep based on the distributions of aftershocks. McNamara et al. [2015] estimated the rupture dimension of Event B to be much larger, perhaps as large as 20 km long by 10 km deep.

For our analysis, we modeled Faults A and B as two-dimensional vertical structures within a strike-slip stress regime. The faults were centered at a depth of 5 km, and each had a height of 2.5 km in the vertical direction. The principal stress magnitudes and fluid pressure initially were $\sigma_{ho} = 150$ MPa, $\sigma_{hn} = 75$ MPa, and...
$p_0 = 45 \text{ MPa}$. Stress and pressure were assumed constant along the vertical dimension of the faults, which is a limitation of our two-dimensional model. Fault A represented a segment along the main Wilzetta Fault and was modeled as a 2.8 km long segment oriented at N35°E. Fault B represented a splay branch off of the main Wilzetta Fault and was modeled as a 4.5 km long segment oriented at N55°E. The interpretation provided by McNamara et al. [2015] suggested that Fault B is likely much longer, but because we were only interested in the nucleation phase of Event B and not the full extent of the rupture, this did not affect our results significantly. The orientation of $\sigma_H$ was N85°E, based on the stress indicators provided by Alt and Zoback [2014] and Walsh and Zoback [2016].

3. Numerical Model

We performed numerical experiments using a reservoir model that coupled transient fluid flow, fault mechanics, and rate-and-state earthquake rupture. Details of the numerical formulation can be found in McClure and Horne [2011], McClure [2015], and Norbeck et al. [2016a, 2016b]. The model assumed that the faults were saturated with water and were surrounded by impermeable basement rock. A quasi-dynamic elasticity formulation was used to model stress transfer along the faults and in the surrounding rock. A two-dimensional displacement discontinuity method was used to relate changes in stress to fault displacement [Bradley, 2014]. The elastic properties of the rock surrounding the faults were assumed to be homogeneous.

Although our model is able to solve the full set of equations that describe poroelasticity in a continuum sense [e.g., see Norbeck and Horne, 2015a and Norbeck and Horne, 2016], it is important to note that in this study we only considered flow within the fault zone structures. Mass transfer between the faults and the surrounding rock and any associated poroelastic deformation of the host rock was neglected based on the assumption of extremely low permeability of basement rock. Nonetheless, our numerical experiments had the character of poroelasticity due to our choice of a nonlinear fault stiffness constitutive relationship between void volume and changes in effective stress. We use the term hydromechanical deformation to describe the faulting process in order to avoid confusion with more general poroelastic treatments.

The two-dimensional representation of the fault structures was a major simplification in the model that has several important consequences when interpreting the results of our analysis. Both faults were vertical, located at the same depth, and had the same height. Stress and pressure gradients in the vertical direction were neglected. The sense of slip during both major events was predominantly strike-slip which justified the use of vertical faults, but in reality geometrical effects would have contributed to the magnitude and shape of the stress changes along Fault B. Our fault model geometry was further justified by the inversion performed by Sun and Hartzell [2014], which placed the hypocenter of the nucleation site of Event B at 5 km depth (similar to the foreshock), and because the distribution of aftershocks on both faults existed at similar depths over the range of several kilometers. Using a two-dimensional model allowed us to employ a spatial discretization along the faults that was fine enough to ensure numerically converged solutions for the earthquake nucleation process for values of the characteristic slip-weakening distance as low as $\delta_c = 50 \times 10^{-6}$ m [Lapusta, 2001; Rice and Ben-Zion, 1996].

3.1. Fluid Flow Along Faults

For one-dimensional Darcy flow in the along-fault direction, fluid mass balance can be expressed as [Norbeck et al., 2016a]

$$\frac{\partial}{\partial x} \left( T \frac{\rho}{\mu} \frac{\partial p}{\partial x} \right) = \frac{\partial}{\partial t} \left( \rho E \right),$$

where $T$ is fault transmissivity, $\rho$ is water density, $\mu$ is water viscosity, and $E$ is fault void aperture. The fault transmissivity can be thought of as the product of permeability and hydraulic aperture (thickness available for flow in the along-fault direction). The void aperture is related to the fault porosity and in our model was assumed to behave according to a nonlinear fault stiffness relationship [Bandis et al., 1983; Barton et al., 1985; Willis-Richards et al., 1996]:

$$E = \frac{E_*}{1 + 9 \left( \frac{\sigma_H}{\sigma_0} \right)},$$
where $\sigma = \sigma - p$ is the effective normal stress, and the constants $E_*$ and $\sigma_*$ define the fault stiffness. Hydromechanical coupling arises entirely from the relationship between void volume and effective stress in equation (3).

### 3.2. Fault Mechanics

For the mode II plane strain quasi-dynamic elasticity formulation and assuming a Mohr-Coulomb-type shear failure criterion, mechanical equilibrium along the fault can be described as [Ben-Zion and Rice, 1997]

$$\tau_0 - \eta V + \Phi = f \sigma + s.$$  \hfill (4)

where $\tau_0$ is the initial shear stress due to the tectonic loading, $V$ is the sliding velocity, $\Phi$ is the quasi-static stress transfer, $\eta$ is a material property related to the shear wave speed of the host rock called the inertial damping parameter, and $s$ is fault cohesion. The friction coefficient was modeled using a rate- and state-dependent constitutive formulation [Dieterich, 1992; Rice et al., 2001; Scholz, 1998]:

$$f(V, \Psi) = a \ln \frac{V}{V_*} + \Psi.$$  \hfill (5)

Here $a$ controls the magnitude of the direct velocity-strengthening effect and $\Psi$ is the state variable. State was assumed to evolve according to the aging law description [Rojas et al., 2009]:

$$\frac{\partial \Psi}{\partial t} = -bV \frac{\delta c}{\delta c} \left\{ 1 - \exp \left[ -\frac{f(V, \Psi) - f_s(V)}{b} \right] \right\}.$$  \hfill (6)

State evolution occurs over a characteristic slip-weakening distance, $\delta_c$, and evolves toward a steady state while sliding at constant $V$. The steady state friction coefficient is defined as

$$f_s(V) = f_* - (b - a) \ln \frac{V}{V_*}.$$  \hfill (7)

In equations (6) and (7), $b$ controls the magnitude of the state evolution effect. Linker and Dieterich [1992] demonstrated through laboratory experiments that $\partial \Psi / \partial t$ may also have a dependence on changes in $\bar{\sigma}$. We neglected that effect in this study, but it may be worthwhile to investigate in future studies. Equations (2) and (4) were solved numerically in a fully coupled framework.

### 4. Theoretical Framework

In this section, we present a brief description of the relevant physical processes considered in our numerical experiments. The range of each model parameter tested in the study was guided by this theoretical framework.

#### 4.1. Static Stress Change Caused by Event A

We modeled Event A as an $M_w$ 4.8 event resulting from a uniform stress drop of roughly 1.7 MPa over a 2.8 km by 2.5 km rupture surface. The spatial distributions of the Coulomb stress changes induced by Event A (as resolved along Fault B) are shown in Figures 2 and 3. The largest changes in Coulomb stress did not occur near the epicenter of Event A but rather near the crack tips of the rupture patch. The distribution of the induced stresses resolved along Fault B are shown in Figure 4. The stress concentrations were large in the vicinity of the tip of Fault A but fell off to the background levels over distances of several hundred meters. The largest induced shear stress and largest reduction in normal stress were both on the order of 2–3 MPa. Based on the initial state of stress and a static friction coefficient of $f_* \approx 0.7$, the critical change in Coulomb stress to initiate slip, $\Delta \sigma_{C, crit} = f_0 \bar{\sigma}_0 + s - \tau_0$, was calculated to be 2.15 MPa.

#### 4.2. Undrained Response and Fault Compliance

Following Segall [2010], the change in fluid mass in a fault zone control volume in response to perturbations $\Delta p$ and $\Delta \sigma$ is

$$\Delta m = \Delta (\rho E) = E_0 \Delta \rho + \rho_0 \Delta E.$$  \hfill (8)

where $E_0$ and $\rho_0$ are the void aperture and water density at a reference state. Water compressibility is defined as $c_w = (1/\rho)(\partial \rho / \partial p)$. The fault compressibility can be defined as $c_f = (1/E)(\partial E / \partial p) = -(1/E)(\partial E / \partial \sigma)$ and can
Figure 2. The distribution of the static stress change caused by Event A. This figure shows the normal stress component of the Coulomb stress change resolved in the orientation of Fault B. Increased compressive stresses were generated behind the rupture patch, whereas decreased compressive stresses were generated ahead of the rupture patch.

be calculated in terms of our numerical model parameters using the nonlinear fault stiffness relationship described by equation (3). Note that the fault compressibility is related directly to the mode I fault compliance, which is usually defined as $\partial E / \partial \sigma$, so we use the terms interchangeably in this paper.

In the undrained limit, the flux term in equation (2) vanishes so the change in fluid mass content in the fault control volume can be expressed as

$$\Delta m = \rho_0 E_0 \left( c_f + c_w \right) \Delta p - c_f \Delta \sigma \equiv 0. \quad (9)$$

The instantaneous change in fluid pressure in the fault zone with respect to the static stress change caused by a nearby earthquake is

$$B = \frac{\Delta \sigma}{\Delta \sigma} = \frac{c_f}{c_f + c_w}. \quad (10)$$

Figure 3. The distribution of the static stress change caused by Event A. This figure shows the shear stress component of the Coulomb stress change resolved in the orientation of Fault B. A reduction in shear stress occurred behind the rupture patch, and concentrations of increased shear stress occurred ahead of the rupture patch.
Figure 4. Distributions of the changes in shear and normal stresses along Fault B resulting from the elastic stress transfer caused by slip on Fault A.

The parameter $B$ is analogous conceptually to the Skempton coefficient in poroelasticity. Laboratory measurements suggest that the Skempton coefficient for rocks ranges from 0.4 to 1 [Lockner and Stanchits, 2002; Roeloffs, 1996].

In the earthquake aftershock triggering process, the instantaneous undrained response acts to negate changes in normal stress at early time. In Figures 5 and 6 we compare the change in Coulomb stress along Fault B for stiff and compliant responses, respectively. Separating the relative influence of the different terms affecting $\Delta \sigma_c$ in equation (1) illustrates that $\Delta \tau$ dominates the immediate response as $B \to 1$.

It is important to recognize the differences between the hydromechanical model applied in this work and traditional poroelastic theory. We used the displacement discontinuity method to perform fault deformation calculations; therefore, the faults were represented mathematically as infinitely thin surfaces. A limitation of the model is that we were unable to resolve mechanical processes within the fault zone material itself. In poroelastic theory, the fault pore volume deforms subject to changes in mean stress [Segall, 2010]. In contrast, the pore volume of the faults in our model deforms subject to the nonlinear stiffness relation (see equation (3)) which is affected only by the normal stress acting on the fault. Cocco and Rice [2002] investigated poroelastic response to stress changes resulting from earthquakes and demonstrated that if the rigidity of the fault zone material is significantly less than that of the intact rock surrounding the fault, then the change in fluid pressure within the fault zone is dominated by changes in the normal stress component (i.e., it is unnecessary to resolve the mean stress within the fault zone). The argument of rigidity contrast effects proposed by Cocco and Rice [2002] supports the use of constitutive relationships such as the one used in this study. Another simplification in our model is that the empirical nature of equation (3) implicitly assumes that the rock grain compressibility is negligible, which is why equation (10) appears slightly different from that derived by Zimmerman [1991].

Figure 5. Change in Coulomb stress following an instantaneous undrained response along Fault B for a stiff fault with $B = 0.5$. 
The stiffness relationship used in our model (equation (3)) was developed for open fractures or joints, and it is not clear if it is directly applicable to fault structures. Fault zones are often believed to contain a low-permeability, stiff inner core surrounded by a more permeable, more compliant fractured damage zone. McClure et al. [2016] used field data from diagnostic fracture injection tests to constrain the stiffness parameters in equation (3) for hydraulic fractures during fracture closure and observed relatively compliant behavior. In this case, $c_f \gg c_w$ and $B \to 1$. For a narrow damage zone composed of many fractures, the mechanical interactions between the individual fractures results in an overall stiffer response than for a single fracture, so it is likely that the stiffness values provided by McClure et al. [2016] are lower bounds for faults. When $c_f$ is taken as the pore compressibility of intact rock, $B$ can be lower. In our analysis we tested a range of scenarios where $B = 0.1, B = 0.5,$ and $B = 0.9$.

### 4.3. Transient Flow and Fault Transmissivity

Following the arrest of Event A, the total normal stress along Fault B remained fixed so that changes in void volume occurred only from pressure changes. Assuming that the fault transmissivity is constant for small pressure changes and before slip begins to occur, equation (2) can be linearized as

$$\frac{T \frac{\partial^2 p}{\partial x^2}}{\mu} = \frac{E_0 (c_f + c_w)}{\mu} \frac{\partial p}{\partial t}. \quad (11)$$

The hydraulic diffusivity for fluid flow along faults, $D = T / \left[ E_0 \left( c_f + c_w \right) \mu \right]$, can be introduced to simplify the appearance of equation (11):

$$D \frac{\partial^2 p}{\partial x^2} = \frac{\partial p}{\partial t}. \quad (12)$$

Pressure transients will occur across characteristic temporal and length scales ($t_c$ and $x_c$) according to

$$\frac{x_c^2}{t_c} \sim D = \frac{T}{E_0 \left( c_f + c_w \right) \mu}. \quad (13)$$

The functional form of hydraulic diffusivity given in equation (13) is distinct from the traditional form used to analyze flow in porous media and is particularly useful for understanding flow along fault zone structures. In particular, we are able to acknowledge, using this form of $D$, many of the complexities and epistemic uncertainties associated with natural faults. Instead of emphasizing permeability, this form of $D$ emphasizes fault transmissivity in order to recognize that the physical thickness of the fault zone and connectivity of the fractured damage zone can be difficult to measure in practice. The void aperture of the fault zone material may involve contributions from both connected and unconnected fracture porosity as well as porosity of the intact rock.

In our numerical experiments, we assumed that the fluid pressure distribution in the fault was spatially uniform at the initial condition. The instantaneous changes in fluid pressure for faults subjected to changes in normal stress is controlled by the fault compliance, as described by equation (10). In our model, stress concentrations were largest in the vicinity of where the two fault surfaces met because the Event A rupture was
assumed to have arrested at that location based on the distribution of aftershocks [McNamara et al., 2015]. In Figures 2 and 4, it is observed that the changes in normal stress resolved on Fault B transitioned from decreased compression (unclamping) to increased compression (clamping) across the intersection with the Fault A crack tip. In Figure 7, a typical simulation result for the transient pressure response along the fault is illustrated. During the initial undrained response, a significant pressure change was induced along Fault B both ahead of and behind the crack tip of Fault A. Pressure gradients induced fluid flow along the fault, and the pressure eventually equilibrated back toward the initial condition. Meanwhile, the total normal stress along Fault B remained constant, so the changes in fluid pressure over time resulted in a transient loading mechanism that influenced the nucleation process.

4.4. Time to Instability

During the nucleation phase of an earthquake while the sliding velocities are still small, the inertial term in equation (4), $\eta V$, can be neglected. In an idealized spring slider faulting model, the stress drop along the fault can be approximated as the product of the mode II fault stiffness and some average slip quantity, i.e., $\Phi \approx -k \langle \delta \rangle$. Taking the time derivative of both sides of equation (4) gives

$$\frac{\partial \tau}{\partial t} = -k V = \frac{a}{V} \left( \frac{\partial V}{\partial t} + \frac{\partial \Psi}{\partial t} \right) + f \frac{\partial \sigma}{\partial t}.$$  \hspace{1cm} (14)

We can define a characteristic stiffness as $k_c = -f \langle \sigma \rangle / (V \langle \partial \Psi / \partial t \rangle)$. Rearranging equation (14) yields a differential equation that describes the evolution of the sliding velocity:

$$\frac{\partial V}{\partial t} = \left( \frac{k_c - k}{a \sigma} \right)^2 V^2 + f \frac{\partial \sigma}{\partial t}.$$  \hspace{1cm} (15)

We recognize that $k_c \sim (f - f_s)$ is a weak function of $V$ and friction is approximately constant at the static friction level during nucleation (i.e., $f \approx f_s$). In general, $\partial p / \partial t$ is not constant during transient flow, but here we assume it to be so for the sake of obtaining a tractable solution. Following Dieterich [1994], equation (15) can be integrated to solve for $V$:

$$V = \frac{V_0}{t}, \quad \frac{\partial p}{\partial t} = 0,$$  \hspace{1cm} (16)

$$V = \left\{ \frac{1}{V_0} + \frac{f_s}{(k_c - k) \langle \partial \sigma / \partial t \rangle} \right\} \exp \left[ -\left( \frac{f_s}{\langle \partial \sigma / \partial t \rangle} \right) \frac{t}{\left( k_c - k \right) \langle \partial \sigma / \partial t \rangle} \right] - \frac{f_s}{(k_c - k) \langle \partial \sigma / \partial t \rangle}.$$  \hspace{1cm} (17)
A typical profile for the evolution of sliding velocity at the nucleation site on Fault B following the end of the foreshock. The initial stress transfer loading caused a jump in the sliding velocity according to equation (20). Next, a gradual loading from transient flow combined with state evolution effects (see equation (15)) caused a mild increase in velocity. Finally, nucleation became inevitable as rapid acceleration occurred. The black line is the result of a numerical simulation. The blue and red dashed lines show the velocity evolution described by equations (16) and (17), respectively, for the same properties used in the simulation.

Setting $1/V = 0$ in equations (16) and (17) and solving for $t$ yields a good approximation for the time to instability:

$$
t_i = \frac{a\sigma}{(k_c - k)V_0}, \quad \frac{\partial p}{\partial t} = 0, \quad \text{(18)}
$$

$$
t_i = \frac{\sigma}{f_s(\frac{\partial p}{\partial t})} \ln \left[ \frac{f_s(\frac{\partial p}{\partial t})}{(k_c - k)V_0} + 1 \right], \quad \frac{\partial p}{\partial t} \neq 0. \quad \text{(19)}
$$

In Figure 8, we show a typical history of sliding velocity along Fault B following the end of the foreshock obtained from the numerical experiments and compare against the analytical solutions described by equations (16) and (17) calculated using parameters representative of the simulations. As Event A died out, the elastic stress transfer effectively loaded the fault with a step change in Coulomb stress. Dieterich [1994] showed that the velocity response to step changes in normal and shear stress is (here neglecting the Linker and Dieterich [1992] effect)

$$
V = V_0 \exp \left( \frac{\tau}{a\sigma} - \frac{\tau_0}{a\sigma_0} \right). \quad \text{(20)}
$$

This effect acted to increase the sliding velocity to a new value. Following the initial loading by elastic stress transfer, fluid flow along the fault acted as a transient loading mechanism. This contributed to additional velocity increase as described by equation (20), which in turn increased the rate of state evolution. Finally, rapid acceleration occurred. Event B nucleated and began to propagate as a sustained rupture.

5. Numerical Experiments

The purpose of the numerical experiments was to determine whether transient effects related to rate-and-state friction evolution and hydromechanical coupling could plausibly explain the 20 h delay between Events A and B. Both the hydromechanical processes (see equations (10) and 13) and frictional processes (see equations (18) and (19)) involve combinations of parameters that can lead to nonunique interpretations of the behavior. It was necessary to specify a realistic set of parameters around which perturbations were made to investigate the relative influence of each physical property.

The rate-and-state frictional properties used in the numerical experiments were based on laboratory friction experiments performed on granite samples with gouge [Dieterich, 1981; Marone and Kilgore, 1993; Marone, 1998]. It has been recognized that the values of the rate-and-state parameters, $a$ and $b$, do not vary largely across different experiments; however, it has been speculated that the characteristic slip-weakening distance, $\delta_c$, may depend on fault roughness and may be scale dependent [Dieterich, 1981; Marone and Kilgore, 1993].
diffusivity was calculated as $\Delta$ on the modeled Coulomb stress changes illustrated in Figures 5 and 6, the diffusive length scale of interest first-order estimate of $D$ we took law decay with distance away from the slip plane. So the estimate of $D$ is roughly 4 times lower than our first-order estimate for the Prague fault. Noting that the field measurements are the dominant triggering mechanism. The characteristic diffusion time of interest was the hydraulic diffusivity of the fault was estimated based on the assumption that hydromechanical effects structure that are capable of hosting large earthquakes. To arrive at a first estimate of the hydraulic properties, we varied transmissivity over 6 orders of magnitude ranging $10^0 \leq$ Ti $\leq 10^6$ m s$^{-1}$. Based on the modeled Coulomb stress changes illustrated in Figures 5 and 6, the diffusive length scale of interest (i.e., the distance over which $\Delta \sigma_c > \Delta \sigma_{c,ini}$) was approximately $x_c \approx 100$ m. A first-order estimate of hydraulic diffusivity was calculated as $D \sim x_c^2 / t_i \approx 0.1$ m$^2$ s$^{-1}$. Shapiro et al. [2005] estimated the hydraulic diffusivity of a fractured granite geothermal reservoir to be $D_0=0.16$ m$^2$ s$^{-1}$, which is the same order of magnitude as our first-order estimate for the Prague fault. Xue et al. [2013] measured the hydraulic properties of a large fault structure based on tidal fluctuations in a borehole and estimated the diffusivity to be $D = 0.024$ m$^2$ s$^{-1}$, which is roughly 4 times lower than our first-order estimate for the Prague fault. Note that the field measurements performed by Xue et al. [2013] did not interrogate the highly damaged zone close to the major slip plane directly but rather a broader damage zone encompassing several hundred meters surrounding the major slip plane, so their estimate of $D$ may be a lower bound. Compared to measurements of fault zone diffusivity, our first-order estimate of $D$ provided a reasonable basis around which to vary parameters in this study.

The hydraulic diffusivity is the ratio of transmissivity to storativity (i.e., $D = T / S = T / \left[ E_0 (c_f + c_w) \mu \right]$). To arrive at a first-order estimate of the storativity of the fault zone, we assumed that most of the storage volume was located within the densely fractured damage zone, because fracture density typically exhibits a power law decay with distance away from the slip plane [Faulkner et al., 2010]. Assuming a 5 m thick damage zone with a porosity of 1% gives an estimate for the fault void volume as $E_0 = 0.05$ m. This value of $E_0$ was held constant in each numerical experiment. The fault zone compressibility influences the storativity of the fault as well as the magnitude of the undrained pressure response. The compressibility of porous rock can vary from an order of magnitude less than water to an order of magnitude greater than water [Horne, 1995; Townend and Zoback, 2000]. As described in section 4.2, the compressibility of individual fractures can be several orders of magnitude larger than water [McClure et al., 2016]. As a first-order estimate of the fault zone compressibility, we took $c_f = c_w$. In the numerical simulations, we tested variable fault compressibility values over the range of $c_w / 9 \leq c_f \leq 9c_w$, which corresponded to Skempton coefficients ranging 0.1 $\leq$ $B \leq 0.9$. Fluid properties were taken to reflect those of water at 45 MPa and 150°C [Lemmon et al., 2016], giving $c_w = 5 \times 10^{-10}$ Pa$^{-1}$, $\rho_w = 940.3$ kg m$^{-3}$, and $\mu = 2 \times 10^{-4}$ Pa s. Based on these values, first-order estimates of fault storativity and transmissivity were calculated to be $S = 10^{-14}$ m s and $T = 10^{-15}$ m$^3$, respectively. In the numerical experiments, we varied transmissivity over 6 orders of magnitude ranging $10^{-18} \leq T \leq 10^{-12}$ m$^3$.

The slip-weakening distance was fixed to a value of $\delta_c = 50 \times 10^{-6}$ m, consistent with laboratory friction measurements [Dieterich, 1981]. The initial value of the state variable, $\Psi_0$, was defined by the initial sliding velocity and the initial stress conditions along the fault to be $\Psi_0 = \tau_0 / \sigma_0 - a \ln (V_0 / V_1)$. One of the primary goals of this work was to develop constraints on the in situ hydraulic properties of fault structures that are capable of hosting large earthquakes. To arrive at a first estimate of the hydraulic properties, the hydraulic diffusivity of the fault was estimated based on the assumption that hydromechanical effects were the dominant triggering mechanism. The characteristic diffusion time of interest was $t_i = 1$ day. Based on the modeled Coulomb stress changes illustrated in Figures 5 and 6, the diffusive length scale of interest (i.e., the distance over which $\Delta \sigma_c > \Delta \sigma_{c,ini}$) was approximately $x_c \approx 100$ m. A first-order estimate of hydraulic diffusivity was calculated as $D \sim x_c^2 / t_i \approx 0.1$ m$^2$ s$^{-1}$. Shapiro et al. [2005] estimated the hydraulic diffusivity of a fractured granite geothermal reservoir to be $D_0=0.16$ m$^2$ s$^{-1}$, which is the same order of magnitude as our first-order estimate for the Prague fault. Xue et al. [2013] measured the hydraulic properties of a large fault structure based on tidal fluctuations in a borehole and estimated the diffusivity to be $D = 0.024$ m$^2$ s$^{-1}$, which is roughly 4 times lower than our first-order estimate for the Prague fault. Note that the field measurements performed by Xue et al. [2013] did not interrogate the highly damaged zone close to the major slip plane directly but rather a broader damage zone encompassing several hundred meters surrounding the major slip plane, so their estimate of $D$ may be a lower bound. Compared to measurements of fault zone diffusivity, our first-order estimate of $D$ provided a reasonable basis around which to vary parameters in this study.

### Table 1. Description of the Four Sets of Numerical Experiments

<table>
<thead>
<tr>
<th>Description</th>
<th>Case 1</th>
<th>Case 2a</th>
<th>Case 2b</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmissivity held constant ($T = 10^{-15}$ m$^3$)</td>
<td>Variable transmissivity ($10^{-18}$ m$^3 \leq T \leq 10^{-12}$ m$^3$)</td>
<td>Variable transmissivity ($10^{-18}$ m$^3 \leq T \leq 10^{-12}$ m$^3$)</td>
<td>Variable direct-effect parameter ($0.005 \leq a \leq 0.02$)</td>
<td></td>
</tr>
<tr>
<td>Relatively stiff fault ($B = 0.5$)</td>
<td>Relatively compliant fault ($B = 0.9$)</td>
<td>Negligible hydromechanical effects ($B = 0$ and $T = 0$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the expression for time to instability (see equation (18)), the time to instability scales with the direct-effect parameter as $t_i \sim a$ and with the initial sliding velocity as $t_i \sim 1 / \Psi_0$. In our experiments, we tested a range of $0.005 \leq a \leq 0.02$ while holding $(b - a) = 0.004$ constant. The initial sliding velocity was held the same in each simulation and was prescribed to be $V_0 = 10^{-12}$ m s$^{-1}$. This value was relatively arbitrary because there are no good constraints on the background sliding velocity of inactive faults in Oklahoma, other than that they must be sliding very slowly given that there have not been any large earthquakes in recent history.
Table 2. Model Properties Used in the Numerical Experiments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>5</td>
<td>km</td>
</tr>
<tr>
<td>$\sigma_{\text{HD}}$</td>
<td>150</td>
<td>MPa</td>
</tr>
<tr>
<td>$\sigma_{\text{H0}}$</td>
<td>75</td>
<td>MPa</td>
</tr>
<tr>
<td>$\sigma_{\text{D0}}$</td>
<td>45</td>
<td>MPa</td>
</tr>
<tr>
<td>$p_0$</td>
<td>45</td>
<td>MPa</td>
</tr>
<tr>
<td>$\theta_{\text{H}}$</td>
<td>N85°E</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_{\text{A}}$</td>
<td>N35°E</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_{\text{B}}$</td>
<td>N55°E</td>
<td>-</td>
</tr>
<tr>
<td>$\mu$</td>
<td>2 × 10^{-4}</td>
<td>Pa s</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>940.3</td>
<td>kg m^{-3}</td>
</tr>
<tr>
<td>$c_w$</td>
<td>$5 \times 10^{-10}$</td>
<td>Pa^{-1}</td>
</tr>
<tr>
<td>$G$</td>
<td>15</td>
<td>GPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>$\eta$</td>
<td>3</td>
<td>MPa s m^{-1}</td>
</tr>
<tr>
<td>$E_0$</td>
<td>0.05</td>
<td>m</td>
</tr>
<tr>
<td>$c_t$</td>
<td>$c_w/9 - 9c_w$</td>
<td>Pa^{-1}</td>
</tr>
<tr>
<td>$T$</td>
<td>$10^{-18} - 10^{-12}$</td>
<td>m³</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.7</td>
<td>-</td>
</tr>
<tr>
<td>$V_*$</td>
<td>$1 \times 10^{-12}$</td>
<td>m s^{-1}</td>
</tr>
<tr>
<td>$a$</td>
<td>0.005 - 0.02</td>
<td>-</td>
</tr>
<tr>
<td>$(b - a)$</td>
<td>0.004</td>
<td>-</td>
</tr>
<tr>
<td>$s$</td>
<td>0.5</td>
<td>MPa</td>
</tr>
</tbody>
</table>

We tested three different scenarios (Cases 1–3) to determine which types of physical processes and groups of fault properties could explain the 1 day delayed triggering between Events A and B (see Table 1 for a description of each case). The model properties used in the numerical experiments are listed in Table 2. The main comparison metric was the time to instability, $t_i$, of Event B.

5.1. Case 1: Fault Compliance

The compressibility or compliance of the fault zone material influenced the magnitude of the undrained loading response to changes in normal stress. In equation (10), larger $c_t$ and larger $B$ allow for larger pressure perturbations in the undrained loading response. Because the pressure perturbations act to negate the changes in total normal stress, this mechanism dampens the initial Coulomb stress change and tends to discourage shear failure. In Case 1, we investigated the influence of fault zone compliance on the time

Figure 9. Sliding velocity history at the Event B nucleation site for Case 1. The actual timing of Event B is shown as the vertical dashed line. The fault zone compressibility was varied from $c_w/9 \leq c_t \leq 9c_w$, which corresponded to Skempton coefficients that varied over the range of $0.1 \leq B \leq 0.9$. For larger fault compressibility, the undrained loading effect that acted to dampen the Coulomb stress change temporarily was more pronounced, which increased the time to instability. For the lowest fault compressibility tested, the time to instability was underestimated by over an order of magnitude.

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to instability of the main earthquake (Event B). The fault zone compressibility was varied over the range of \( c_w / 9 \leq c_f \leq 9c_w \), which corresponded to Skempton coefficient values ranging \( 0.1 \leq B \leq 0.9 \). In each case, the fault zone transmissivity was held constant at \( T = 10^{-15} \) m$^3$.

In Figure 9, the sliding velocity histories at the Event B nucleation site for Case 1 are shown. The fault zone compliance affected the time to instability most notably through the initial rise in sliding velocity, which was influenced by the magnitude of \( \Delta \sigma_C \), experienced by Fault B during the foreshock. For the fault with the lowest compliance \( (B = 0.1) \), nearly the full static stress change was realized immediately, driving the fault to rupture only 52 min following the arrest of Event A. For the scenarios where \( B = 0.5 \) and \( B = 0.9 \), the effect of the Coulomb stress dampening was to delay the onset of instability, which resulted in times to instability of 3.5 h and 14.3 h, respectively. The results of this numerical experiment demonstrate that hydromechanical coupling, in particular, the undrained loading response, can influence the earthquake nucleation process significantly.

5.2. Case 2: Fault Transmissivity

In our numerical experiments, the undrained loading response resulted in significant fluid pressure perturbations within the fault zone. Fluid flow occurred along the fault structure in response to the pressure gradients that developed, ultimately driving the fluid pressure distribution in the fault back toward equilibrium. Because the change in total normal stress caused by the foreshock remained constant, the transient flow period acted as a loading mechanism by gradually reducing the effective normal stress acting over some parts of the fault (see Figure 7). The transmissivity of the fault controlled the time scales over which the transient loading occurred. Laboratory and field experiments have indicated that fault transmissivity can vary over many
orders of magnitude. In Case 2, we performed two sets of numerical experiments to investigate the effect of the fault transmissivity by varying transmissivity over the range of $10^{-18} \leq T \leq 10^{-12}$ m$^3$. In Case 2a relatively stiff behavior was tested ($B = 0.5$), and in Case 2b relatively compliant behavior was tested ($B = 0.9$).

The sliding velocity profiles for Cases 2a and 2b are shown in Figures 10 and 11, respectively. In both cases, the scenarios with the largest transmissivity underestimated the time to instability by nearly 2 orders of magnitude. The scenarios with medium and low transmissivity estimated the time to instability within the same order of magnitude compared to the actual event timing. The combination of fault properties that resulted in the closest match was characterized by relatively high compliance ($B = 0.9$) and low to medium transmissivity ($T = 10^{-18}$ to $T = 10^{-15}$ m$^3$).

5.3. Case 3: Neglecting Hydromechanical Coupling

The rate-and-state friction framework involves many fault properties that bear substantial uncertainty. In Case 3, we investigated whether the timing of the main shock could be described purely by rate-and-state effects by neglecting hydromechanical coupling completely. By neglecting the undrained loading effect, the full static stress change was realized instantaneously. In this set of numerical experiments, the fault hydraulic properties were $B = 0$ and $T = 0$, and the rate-and-state direct-effect parameter was varied over the range of $0.005 \leq a \leq 0.02$. The value of $a$ influenced the time to instability predominantly by affecting the magnitude of the initial rise in sliding velocity and also through state evolution effects.

The sliding velocity profiles for Case 3 are shown in Figure 12. Each of the scenarios tested exhibited the same qualitative behavior. An initial jump in sliding velocity was followed by an extended period of time where velocity remained relatively constant. In contrast to the scenarios in Case 2 that showed strong transient loading effects, the onset of rapid acceleration occurred in a manner predicted by equation (18). In this set of experiments, a significant variation in $t_i$ was observed, with values ranging from $t_i = 2$ min (for the scenario with $a = 0.005$) to $t_i = 5$ days (for the scenario with $a = 0.02$). The predominant influence of $a$ was on the magnitude of the rise in sliding velocity caused by the static stress change (see equation (20)). The value of $a$ also influenced the time to instability through state evolution effects (see equation (18)). Interpolating between these results, it is evident that using a value for the direct-effect parameter between $a = 0.01$ and $a = 0.02$ would have been able to match the 20 h delay period.

6. Discussion

It is evident from equations (2)–(6) that stress and friction along a fault can be influenced by transient processes including both fluid flow and state evolution. Using a simple “static” description of fault friction, a poroelastic response and subsequent pressure relaxation provide a physics-based explanation of delayed triggering and aftershock decay [Cocco and Rice, 2002; Nur and Booker, 1972; Roeloffs, 1996]. Using a rate-and-state description of friction, Dieterich [1994] investigated how step changes in shear and normal stress can influence the earthquake nucleation process. The results of that study demonstrated a delayed triggering process between earthquakes and subsequent aftershocks even for loading that remained constant in time.
Segall and Lu [2015] extended that analysis to incorporate the effects of transient fluid flow and associated poroelastic stressing in the earthquake nucleation process in a semianalytical model. In this study, we sought to identify whether hydromechanical effects were necessary to explain the 20 h delay between Events A and B in the Prague earthquake sequence and, in doing so, learn about the in situ properties of basement faults in Oklahoma. The study was informed by previous work that enabled characterization of the stressorientations, reservoir fluid pressure, fault geometry, and earthquake source mechanisms [Alt and Zoback, 2014; Keranen et al., 2013; McNamara et al., 2015; Nelson et al., 2015; Sumy et al., 2014; Sun and Hartzell, 2014]. Estimates of the stress magnitudes at depth in the area near Prague, Oklahoma, were provided by Walsh and Zoback [2016]. Assimilation of these sources of information enabled accurate modeling of the stress change caused by Event A resolved along Fault B and provided constraints on Fault B’s initial proximity to failure. We performed experiments using a fully coupled fluid flow and fault mechanics numerical model that performed earthquake rupture calculations within the context of rate-and-state friction.

The results of our numerical experiments suggested that several different scenarios could explain the delayed triggering. This reflects the nonunique combinations of parameters that can influence earthquake nucleation. The exact nature of the physical processes that led to the delay between the foreshock and main shock at Prague remain ambiguous. Nonetheless, this study demonstrated that for specific circumstances it was possible to set useful bounds on fault transmissivity, fault compliance, and rate-and-state frictional properties. Here we review the major conclusions drawn from our study.

In Cases 1 and 2, we investigated the roles of various hydromechanical processes in the earthquake nucleation process, including pressure changes during undrained loading conditions and transient changes in effective stress caused by flow along faults. In our numerical experiments, varying the hydraulic properties of the fault influenced the timing of Event B significantly. A competition existed between a damped static stress change caused by fault compliance, which tended to delay rupture, and a loading mechanism caused by transient fluid flow, which tended to encourage rupture. Our results suggested that relatively compliant behavior ($B = 0.9$) combined with medium to low transmissivity ($T = 10^{-18}$ to $10^{-15}$ m$^3$/s) was consistent with the earthquake timing from the field observations.

Comparing Figures 10 and 11, it was observed that the time to instability for the highly transmissive faults was insensitive to fault compliance ($t_i$ in these two scenarios was nearly identical, even though $B$ was significantly different). Although the theory presented in section 4.4 was developed for idealized cases (e.g., constant loading rate), we can apply those principles to aid in the interpretation of the behavior observed in the numerical experiments. In general, step changes in $\Delta \sigma_c$ act to bring the fault to a new “initial” sliding velocity through a rate-strengthening process described by equation (20). For faults with very low transmissivity there should be no transient loading mechanism (i.e., $\dot{p}/\dot{t} = 0$), and the time to instability scales as $t_i \sim V_0^{-1}$ (see equation (18)). However, when the transient loading mechanism is nonnegligible, equation (19) indicates that the transient loading term can dominate ($t_i \sim \partial \sigma / \partial t$), and the influence of the initial sliding velocity is reduced to a logarithmic dependence ($t_i \sim \ln V_0^{-1}$). Recent field experiments, such as the one described by Guglielmi et al. [2015], have equipped wells with strain gauges to measure slip across faults that intersect the wellbore. With this type of technology, it could be possible to measure the evolution of the sliding velocity at the nucleation site of an earthquake. The numerical modeling analysis performed in the present study would be useful for informing future field studies related to injection-induced earthquakes and earthquake nucleation.

In Case 3, we neglected the hydromechanical response altogether, and nucleation was driven purely by state evolution effects. In this set of experiments, the timing of Event B was underestimated by 1 to 3 orders of magnitude (relative to the actual timing observed in the Prague earthquake sequence) for $0.01 \leq a \leq 0.007$ (see Figure 12). For $0.01 \leq a \leq 0.02$, the time to instability was on the same order of magnitude as the field observations. Laboratory friction experiments on granite rock with simulated gouge typically show measurements of the direct-effect parameter ranging $0.005 < a < 0.015$ [Blanpied et al., 1991; Dieterich and Kilgore, 1996; Marone and Kilgore, 1993; Marone, 1998]. The range of $a$ values that led to aftershock timings consistent with the field observations were on the high end of the values measured in the laboratory but were certainly within a realistic range. Thus, the possibility that the nucleation phase of the Prague main shock may have been influenced exclusively by rate-and-state friction effects cannot be precluded.
Further investigation within the context of the conceptual model proposed in this study is justified. In the rate-and-state friction framework, the time to instability for earthquake nucleation is extremely sensitive to the initial sliding velocity prescribed to the fault as well as the state evolution law. In this work, we assumed a relatively arbitrary value for the initial sliding velocity of $V_0 = 10^{-12} \text{ m s}^{-1}$ due to the epistemic uncertainty associated with the background sliding velocity of inactive faults in Oklahoma. It is arguable that the sliding velocity may have been significantly lower based on the observation that there have been no significant earthquakes on this fault in recent geologic history. Alternatively, one may argue that if the fault had been influenced by nearby fluid injection, then it is plausible that the fault was at an elevated state of sliding just prior to the foreshock. We tested only the aging law form of state evolution. It would be worthwhile to test the slip law, in which state can only evolve during sliding. Furthermore, based on the hypothesis that the Prague sequence was induced by fluid injection, application of a form of the slip law that is coupled with the variable normal stress mechanism proposed by Linker and Dieterich [1992] may provide new insight into the manner in which seemingly inactive faults become excited by changes in fluid pressure.

In future studies of the Prague earthquake sequence, it would be worthwhile to investigate alternative conceptual models other than Coulomb stress triggering. For example, if it is assumed that the foreshock and main shock were fluid-injection-induced earthquakes, and then a major inconsistency that must be overcome is that based on the stress measurements, the foreshock likely occurred on a fault segment (Fault A) that was much more poorly oriented for shear failure than the fault that hosted the main shock (Fault B). One possible explanation is that initially Fault A was connected hydraulically to the injection aquifer and Fault B was not connected hydraulically to either Fault A or the injection aquifer. Following the foreshock, a permeable pathway connecting the two faults could have been created allowing pressure to build along Fault B, ultimately triggering the event. Shear-enhanced permeability along faults has been observed during hydraulic stimulation treatments in geothermal reservoirs [Dempsey et al., 2015; Häring et al., 2008] and has also been argued to control fluid migration in natural geologic processes [Sibson, 2014]. The effects of variable fault transmissivity could be investigated by incorporating a constitutive relationship between effective stress, fault slip, and fault transmissivity [Norbeck et al., 2016a].

7. Concluding Remarks

We investigated several potential mechanisms for the 20 h delayed triggering between the $M_w$ 4.8 foreshock and the $M_w$ 5.6 main shock during the 2011 Prague, Oklahoma earthquake sequence. We performed three sets of numerical experiments to isolate the effects of different physical processes, including the instantaneous undrained response to changes in normal stress, transient fluid flow along the fault, and the evolution of the state variable and its influence on fault friction. The purpose of the study was to take advantage of the well-characterized state of stress, fault geometry, and earthquake source mechanisms combined with the observation of the time delay between events in order to set constraints on the in situ properties of basement faults in Oklahoma. The main conclusions drawn from the results of the numerical experiments were as follows:

1. A coupling between transient hydromechanical loading and transient friction evolution can plausibly explain the timing of the main shock.
2. Relatively compliant behavior in response to changes in normal stress, corresponding to a Skempton pore pressure coefficient near 1, was consistent with the field observations.
3. The fault transmissivity was estimated to range from $10^{-18}$ to $10^{-15} \text{ m}^3$.

Due to the nonunique combination of fault properties and stress conditions that influence the earthquake nucleation process, it was not possible to identify the exact physical process that led to the 20 h delay between the foreshock and main shock in the Prague sequence. However, this study provided insight into the coupled interactions between elastic stress transfer, hydromechanical response, and the transient evolution of fault friction that may have occurred during the earthquake sequence. The analysis ultimately yielded useful constraints on in situ fault properties within the context of the hypothesis tested in this study, which has broad implications for understanding faulting behavior for other large-scale basement faults in Oklahoma and the rest of the central and eastern United States if the properties can be extrapolated elsewhere. The range of fault properties inferred from this study will provide a basis for physics-based seismic hazard models of injection-induced seismicity.
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References


